Engineering Problems From the Prospective of Multiphysics

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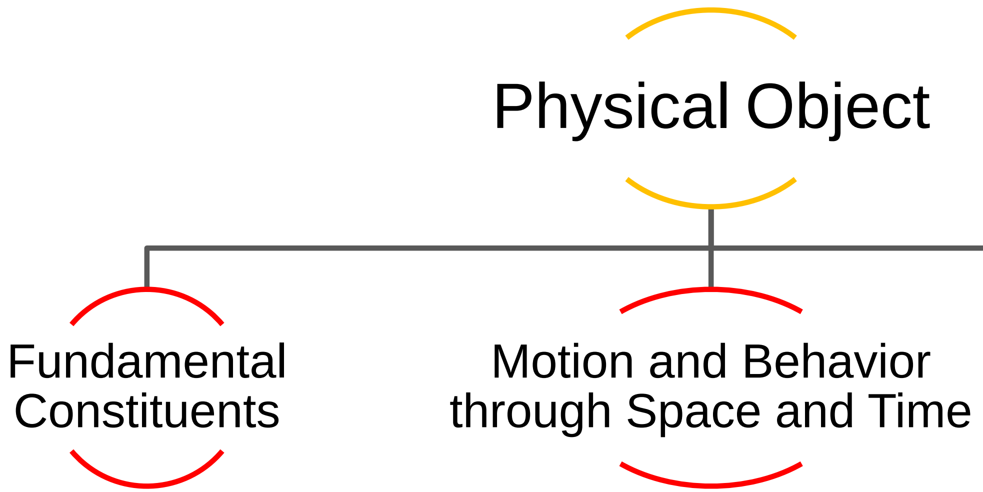
1. Physics

1. Physics

- Known as the natural science of matter, involving study:



Problem facing single-physics analyses

The simultaneity of real events and processes cannot be accounted for

Overcome this problem

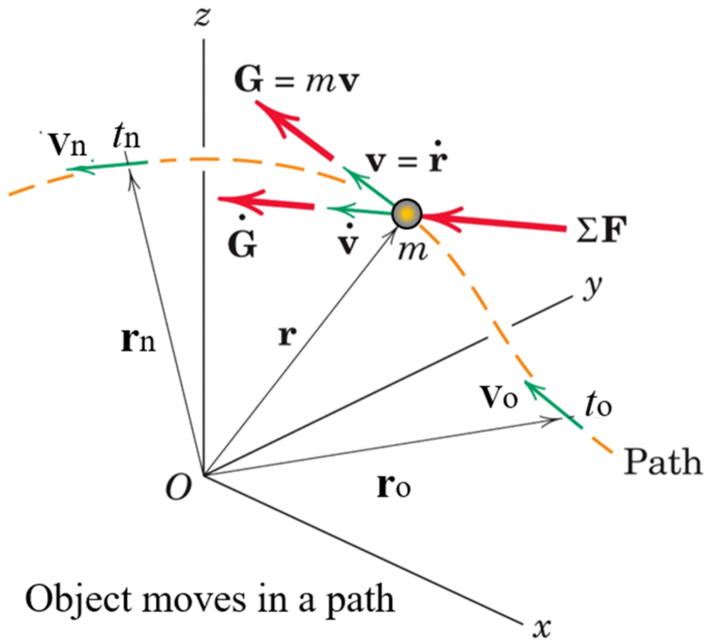
One solution should account for the simultaneity of real events and processes.

Multiphysics

2. Multiphysics

2. Multiphysics

- Defined as the coupled systems involving more than one simultaneously occurring physical field and the studies of knowledge about these systems.
- For example, An object moves according to Newton's second law, will be considered as a Multiphysics problem if the velocity is calculated at every point of the domain to obtain a field.



Object moves in a path

2.1 The Evolution of Multiphysics

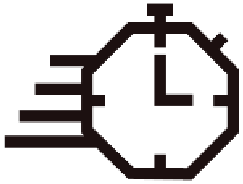
- Concept utilized since the analysis of complex systems began.
- Modern application and understanding enhanced by computational advancements.
- Early computational efforts (1950s-1970s) marked by simple coupled problems.
- 1980s: Birth of commercial Multiphysics software for efficient solutions.
- 1990s-2000s: Increased computational power via Moore's law enabled solving complex problems with finer resolutions.
- 2000s-Present: Expansion and specialization of Multiphysics software, used in diverse applications from electronics to climate modeling.

2.2 Important of Multiphysics

- Interdisciplinary Collaboration
- Realistic Modeling
- Innovation and Progress
- Education and Training
- Software Development
- Addressing Global Challenges
- Future Perspectives

2.3 Challenges in Multiphysics

- Computational Demand: Intensive resource and time requirements.

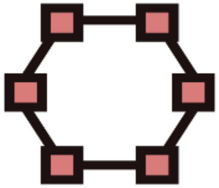


- Modeling Accuracy: Errors in one part can affect the entire system.

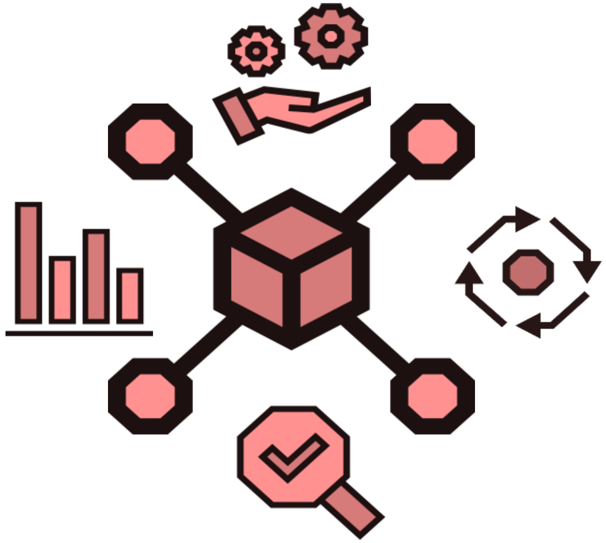
- Software Complexity: Development and effective use can be challenging.

- Validation: Difficulty due to system complexity and lack of data.

- Knowledge Integration: Requires interdisciplinary expertise.



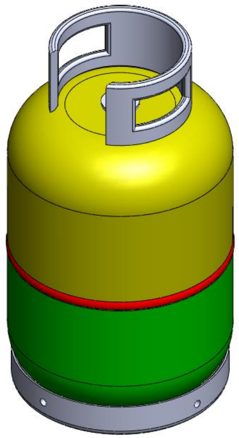
- Today’s problems, Almost all applications have complex systems that involve many distinct physical processes.
- The issue of coupling models of different events at different scales and governed by different physical laws is largely wide open and represents an enormously challenging area for future research.

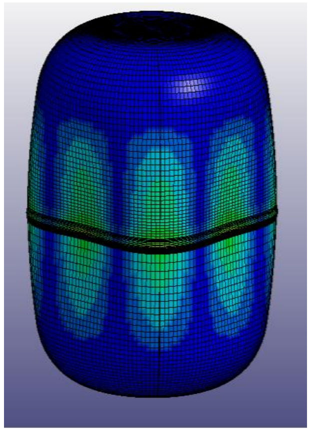


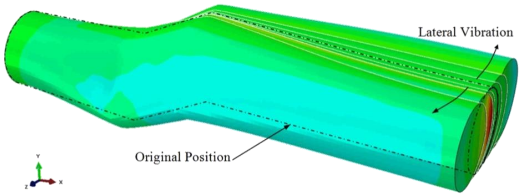
3. Engineering Problems and Multiphysics

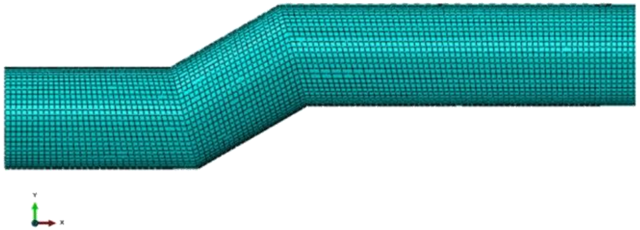
3.1 Introduction

- Majority of engineering solutions are not Multiphysics in nature.
- Problems were simplified to reduce the complexity of the design which also affected the quality and cost of the products
- Mathematical models used in Multiphysics simulations are generally a set of coupled equations.









- Mathematical models can be divided into three categories:

Governing Equation

The equations derived from the physics of the system, which determine the system's characteristics and behaviors. The values of the unknown variables change when one or more of the known variables change.

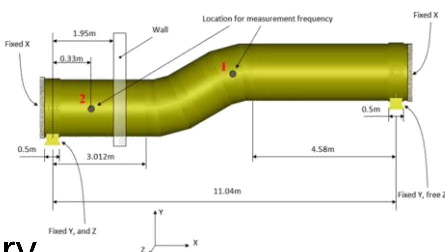
Mathematical Models

Auxiliary Equations

Equations obtained from the standard form of a linear differential equations by replacing the right member by zero.

Boundary/ Initial Condition

Constraints or limitations that are placed on the solutions to differential equations at the boundaries of the domain under consideration



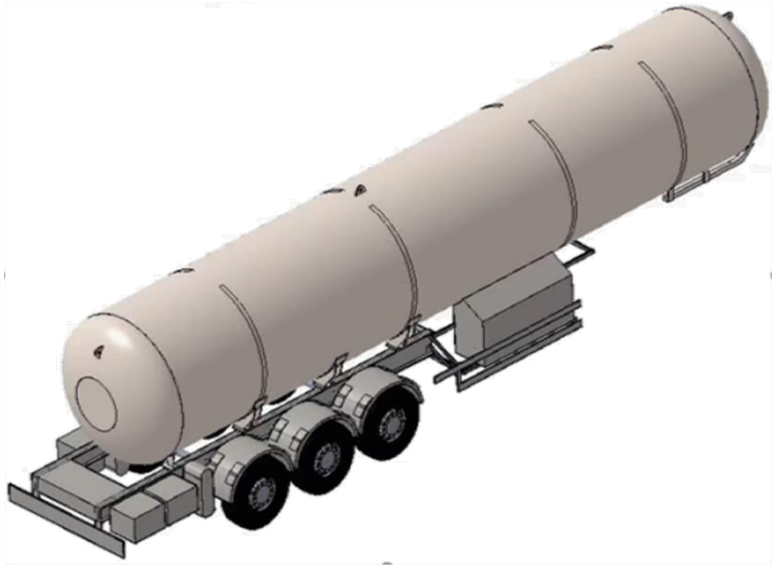
- Dirichlet: Specifies solution's value at the boundary.
- Neumann: Specifies solution's derivative at the boundary.
- Robin: Specifies a relation between the solution's value and its derivative.
- Mixed: Combination of the above over different parts of the boundary.
- Periodic: Requires the solution to be periodic over the domain.

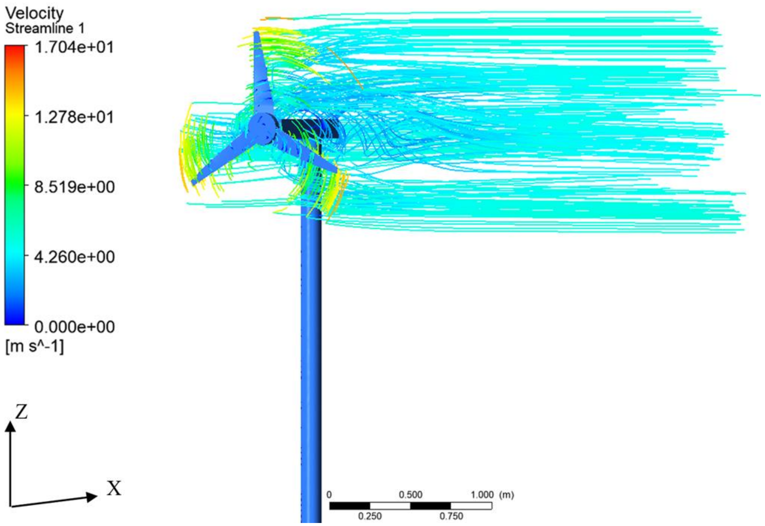
- When using numerical methods or adding boundary conditions (could not exist in physics), the physical complexities are increasingly ignored.
- The treatment is highly dependent on the success of solution methodologies in providing robust and reliable solution algorithms.
- Engineers have recently developed Multiphysics analysis to more accurately represent complex processes' behavior by modeling multiple systems simultaneously.
- Their goal is to improve modeling accuracy and validate the processes by comparing their results with experimental tests.

Enhancement of Multiphysics analyses mainly depends on

- a better understanding of physics and physical representations of interactions
- Ensure complex and large mathematical equations can be solved
- Take into consideration the cost-effectiveness analysis
- Determine the amount of time and space to allocate the necessary modeling complexity

- Finally, in recent years we have seen the engineers using Multiphysics approaches with real-time modelling and are more applicable to address real-world problems.





3.2 Performance Steps for a Multiphysics Simulation

Concept

Real systems are governed by interaction of several physical phenomena in close control boundary

Example

Water Pump involves understanding fluid flow, vibration of the pump, and mechanical stress caused by high pressure.

Solution

It is solved by coupling these different physical models.

Mechanism

The method of the solution could be simultaneously or iteratively depending on the type of coupling and how the physics interact together.

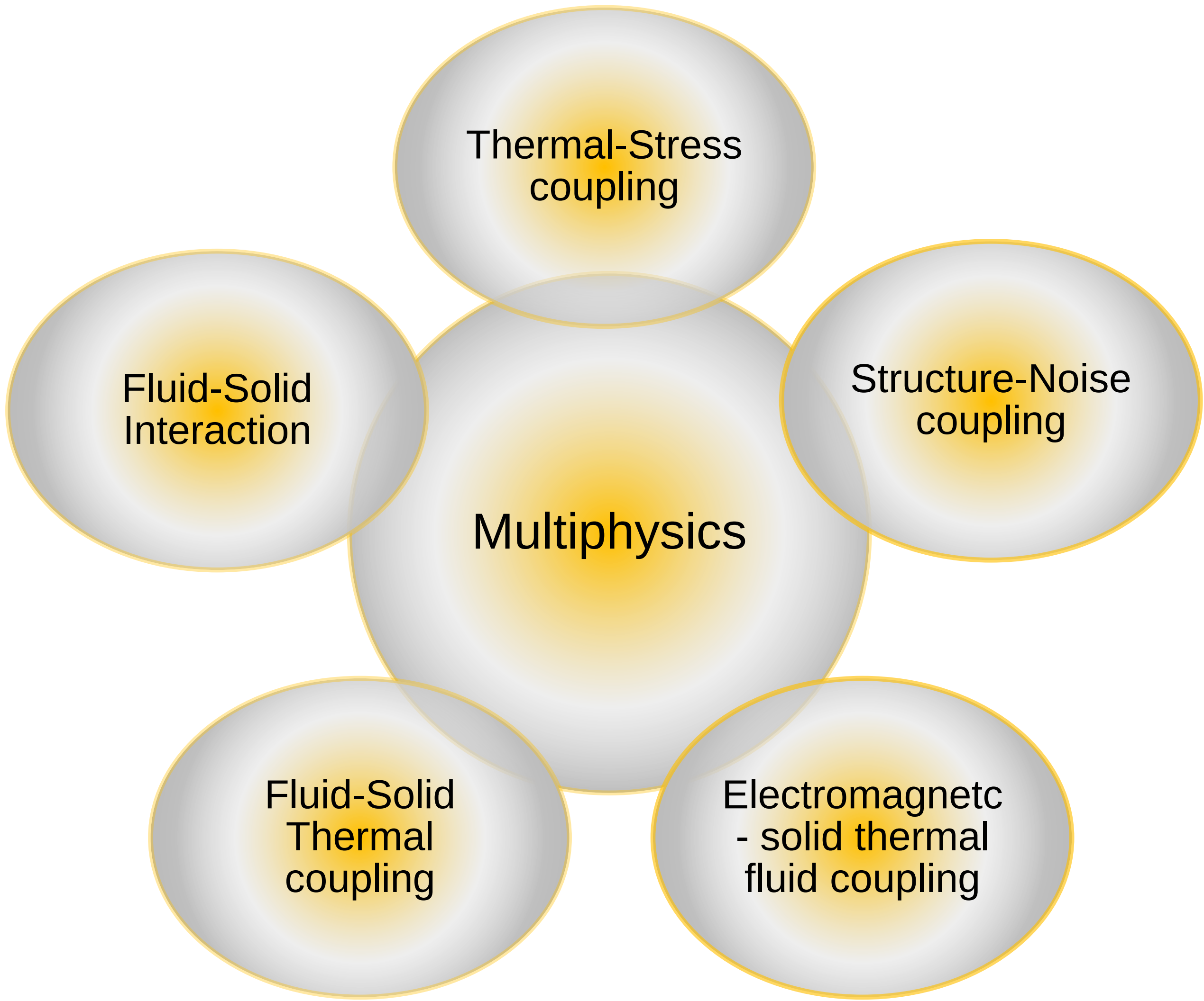
Summary

The complex interdependencies between different physical phenomena are what Multiphysics is about understanding and modeling. To combine these phenomena, one must have a deep understanding of individual phenomena and mathematical tools.

3.3 Field of Multiphysics Applications

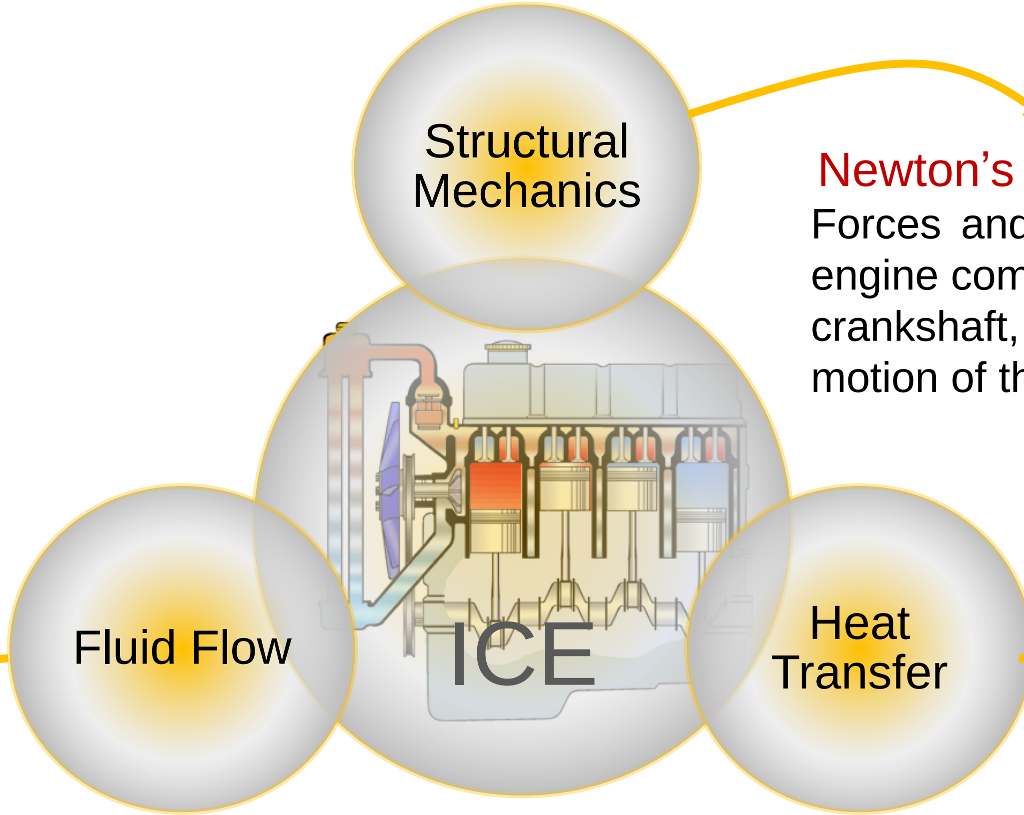
- Every single field in physics can be represented as the following:
  - Heat transfer (thermo- or thermal)
  - Pore water movement (hydro-)
  - Concentration field (concentro or diffuso/convecto/advecto)
  - Stress and strain (mechano- or Solid/Structural)
  - Dynamics (dyno-)
  - Chemical reactions (chemo- or chemico-)
  - Electrostatics (electro-), and magnetostatics (magneto-)
  - Etc…
- In Multiphysics, coupled problem is usually titled using compound words such as: thermo-hydro-mechanical.

- Combinations of the above physics fields can lead to several possible types of Multiphysics.
- However, a type of Multiphysical phenomenon under the above name based on the observations in nature and sciences and the needs for practice is appears.
- Based on a review of Multiphysics with an emphasis in porous materials, the most representative Multiphysical processes are believed to be:



Most Representative Problems in Multiphysics

- Ex., These physical phenomena interact and influence each other as shown e.g. below



Newton's Laws of Motion

Forces and stresses on the engine components (pistons, crankshaft, etc.) due to the motion of the parts

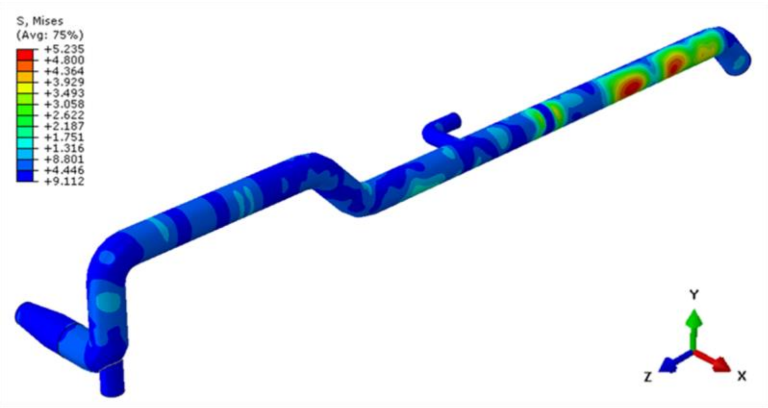
Navier-Stokes Equations

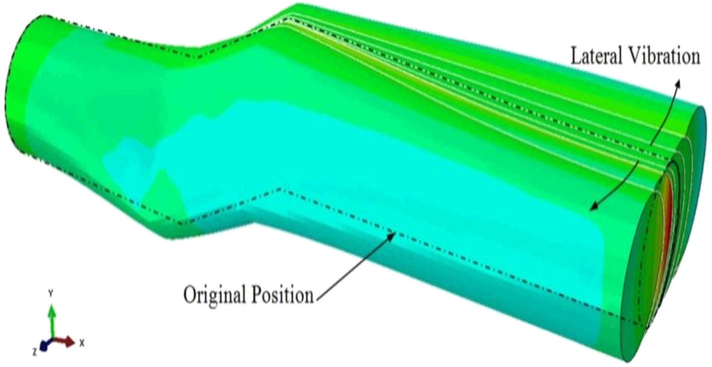
The flow of air and fuel mixture into the cylinders and the exhaust gases out of the cylinders, which impacts the combustion process and engine efficiency.

Heat Conduction and Radiation Equations

The heat generated from the combustion process and friction between moving parts, which can affect the material properties and performance of the engine.

- In real-world systems,
  - a coupling between solid mechanics, and fluid dynamics is often referred to as fluid-structure interaction (FSI).
  - a movement of fluid in a pipe can lead to vibration is referred to a Flow-induced vibration (FIV) which indicate that the vibrations are caused by the interaction of a structural component with a fluid flow surrounding it.
  - stresses and deformation in a solid body under thermal loads is called thermoelastic body.



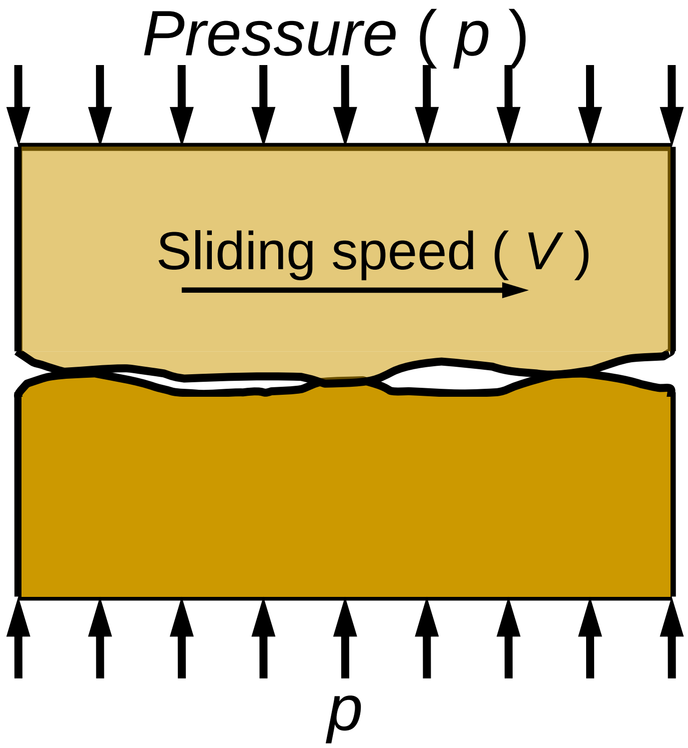


4. Engineering Application in Multiphysics

4.1 Thermoelastic Sliding Bodies

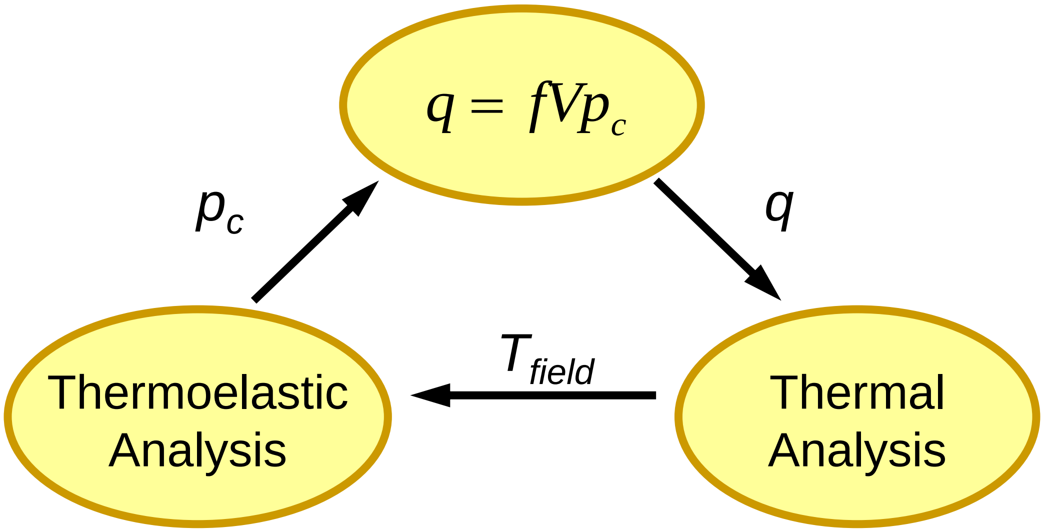
4.1.1 Background

- Frictional heating generated in sliding systems causes thermoelastic distortion
- When frictional heating is large enough between two contact bodies:
  - The disturbance grows
  - The body shape changes
  - Contact pressure is influenced

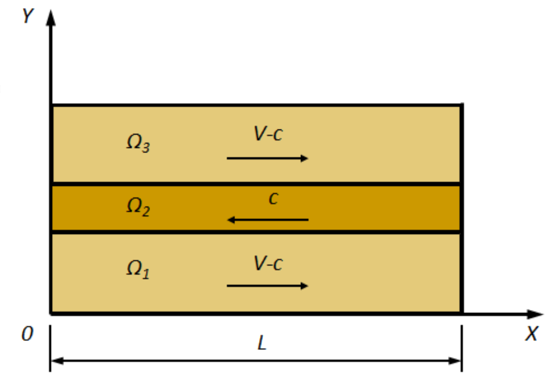


4.1.2 Thermomechanical Coupling Algorithm

- Contact bodies create a thermomechanical coupling problem involving heat conduction and thermoelastic distortion that requires a separate algorithm for each part, combining the results through an iterative method.
- q is the heat flux and p_c is the contact pressure between surface
- This phenomenon is known as Thermoelastic Instability (TEI).



4.1.3 FE Modeling and Assumptions



- A reference system, OXY
- Each layer introduces a migration speed (c) of perturbation with an independent moving frame relative to the reference system frame Ω_1 and Ω_3 move with respect to reference system by constant sliding speed (V-c)
- Ω_2 moves with respect to reference system by constant sliding speed (-c)
- No shear stress at contact region
- An initial contact pressure perturbation pint is applied between surface
- Cyclic boundary condition at both sides with wavelength L=2 π/m (m is the wave number)

4.1.4 Thermal Analysis

- The heat conduction equation for layers Ω_1 and Ω_3:

$$K_i\left(\frac{\partial^2 T_i}{\partial X^2}+\frac{\partial^2 T_i}{\partial Y^2}\right)-c_{p,i}\rho_i(V-c)\frac{\partial T_i}{\partial X}=0, \quad i=1 \text{ or } 3$$

- For layer Ω_2:

$$K_2\left(\frac{\partial^2 T_2}{\partial X^2}+\frac{\partial^2 T_2}{\partial Y^2}\right)-c_{p,2}\rho_2 c\frac{\partial T_2}{\partial X}=0$$

- Where c_p , ρ , and K denote specific heat, density, and conductivity

4.1.5 Thermal boundary conditions

- At the interfaces between the layers, heat is generated due to sliding friction:

$$q_i = fVp_i$$

- where *f* is the coefficient of friction and pi represents the contact pressure at the interface Yi (i = 1; 2)
- The thermal boundary conditions at these interfaces depend on the local contact condition. In regions of contact, we have the thermal balance condition

$$K_i \frac{\partial T_i}{\partial Y}\bigg|_{Y=Y_i} - K_{i+1} \frac{\partial T_{i+1}}{\partial Y}\bigg|_{Y=Y_i} = q_i, \quad i = 1,2$$

$$T_i|_{Y=Y_i} = T_{i+1}|_{Y=Y_i}, \quad i = 1,2$$

Cont. 4.1.5 Thermal boundary conditions

- In regions of separation, there is no heat generated (since pi =0)

$$\left.\frac{\partial T_i}{\partial Y}\right|_{Y=Y_i} = \left.\frac{\partial T_{i+1}}{\partial Y}\right|_{Y=Y_i} = 0, \quad i = 1, 2$$

- For the two exposed edges of the model, we assume that heat is exchanged with the environment through a heat transfer coefficient giving

$$-K_1 \left.\frac{\partial T}{\partial Y}\right|_{Y=0} = \bar{h}(T(X,0) - T_0); \quad K_3 \left.\frac{\partial T}{\partial Y}\right|_{Y=Y_3} = \bar{h}(T(X,Y_3) - T_0)$$

- On the boundaries x=0 and x=L, we apply the cyclic symmetry boundary conditions

$$T_i|_{x=0} = T_i|_{x=L}, \quad i = 1, 2, 3$$

4.1.6 Elastic Analysis

- A standard elastic contact finite element formulation was used that imposes continuity of normal displacements u_y across the contact interfaces if the contact condition is satisfied and otherwise defines the surfaces as traction-free.

4.1.7 Elastic Boundary Conditions

- On the edges x=0 and x=L, cyclic symmetry conditions are imposed

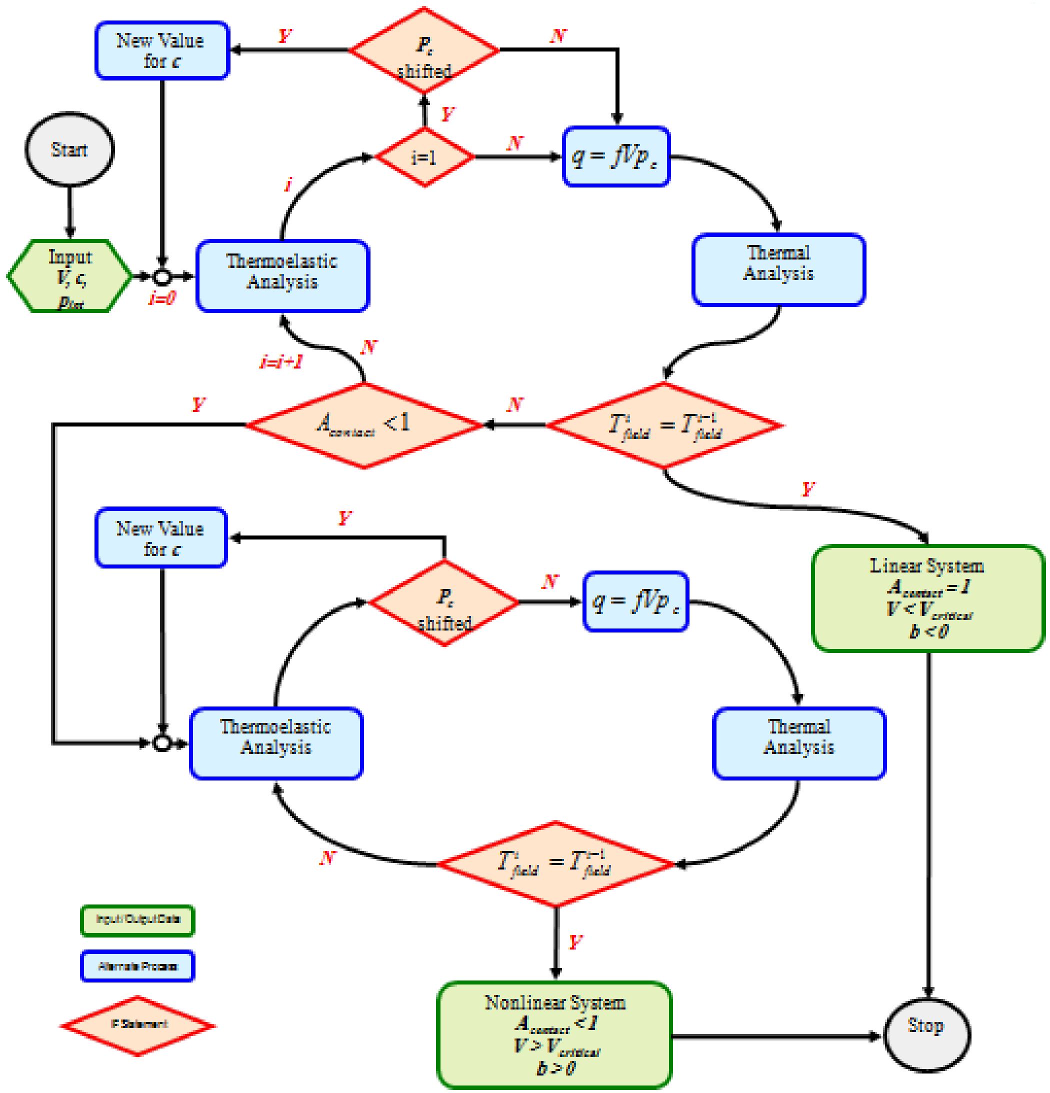
$$u_x|_{X=0} = u_x|_{X=L}, \quad u_y|_{X=0} = u_y|_{X=L}$$

- whereas on the edges Y=0 and Y=Y3, the boundary conditions are:

$$\sigma_{yx}|_{Y=0} = 0, \quad u_y|_{Y=0} = \gamma\sigma_{yx}|_{Y=Y_3} = 0, \quad u_y|_{Y=Y_3} = 0$$

- Where γ is constant
- The conditions are equivalent to the statement that each of these edges is in frictionless contact with a plane rigid body. The constant γ represents a rigid body displacement that is determined from the supplementary condition that the total force, Fy, in the Y-direction is kept constant.

4.1.8 Coupling Algorithm Flowchart



4.1.9 Data Used in the Analysis

- Properties and geometry of layers Ω_1 and Ω_3 and central layer Ω_2

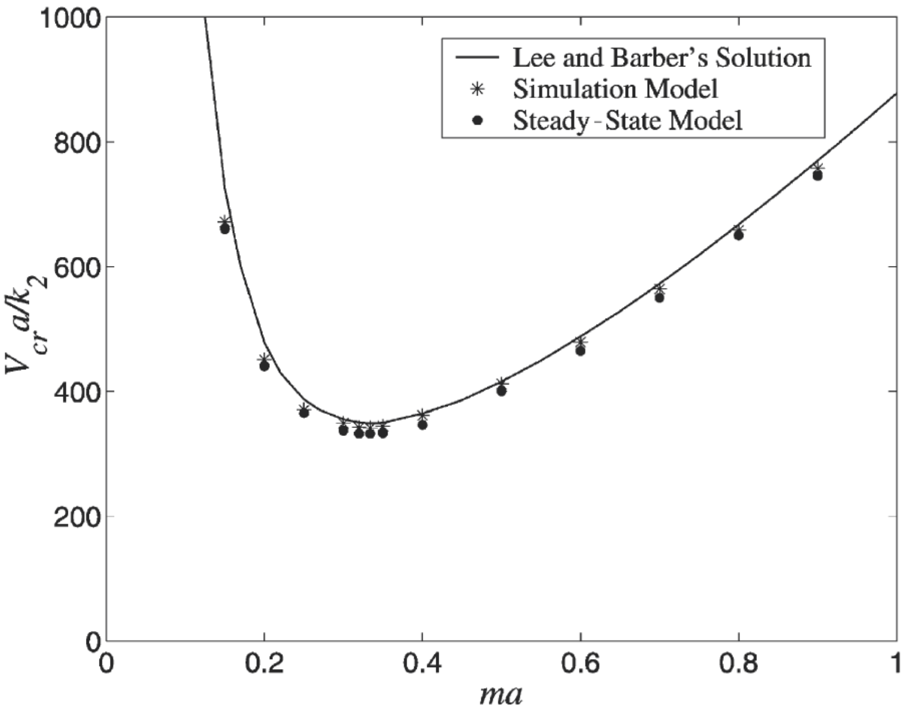
Properties	Ω_2	Ω_1 and Ω_3
Type of material	Steel	Friction
Elastic Modulus, E (GN/m^2)	210	3
Poisson's Ratio, ν	0.494	0.494
Thermal conductivity, K (W/m°C)	42	1
Thermal diffusivity, k (μm^2/s)	11.91	4.167
Thermal expansion, α (μC^{-1})	12	12
Layer thickness, (mm)	$2a = 10$	$H = 50$

- Poisson's ratio in both materials was selected to be close to 0.5, so that the effect of the shear tractions on the elastic contact problem is negligible.
  - This has the advantage of reducing computation time and has very little effect on the results of the analyses because the conductivities of the contact layers are very different.
- The coefficient of friction is f =0.2 and that the ambient air temperature is To =0°C.

4.1.10 Results of Thermoelastic Sliding Body

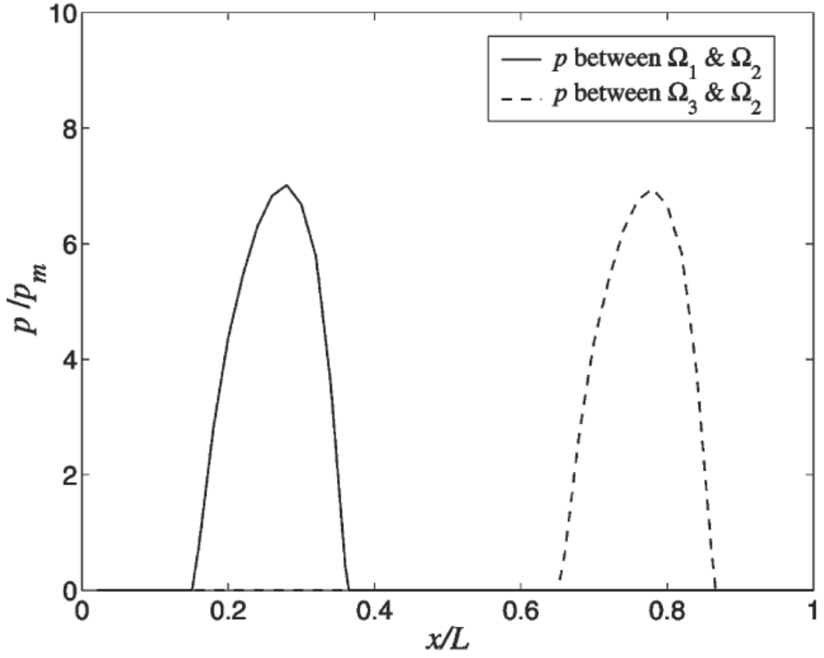
Models Verification

- Both the present steady-state solution and the transient simulation method predict slightly lower critical speeds than the analytical solution
- This discrepancy is probably due to the effect of the finite boundary
- This difference is never more than 6% and the present solution agrees with the transient simulation within 2%.



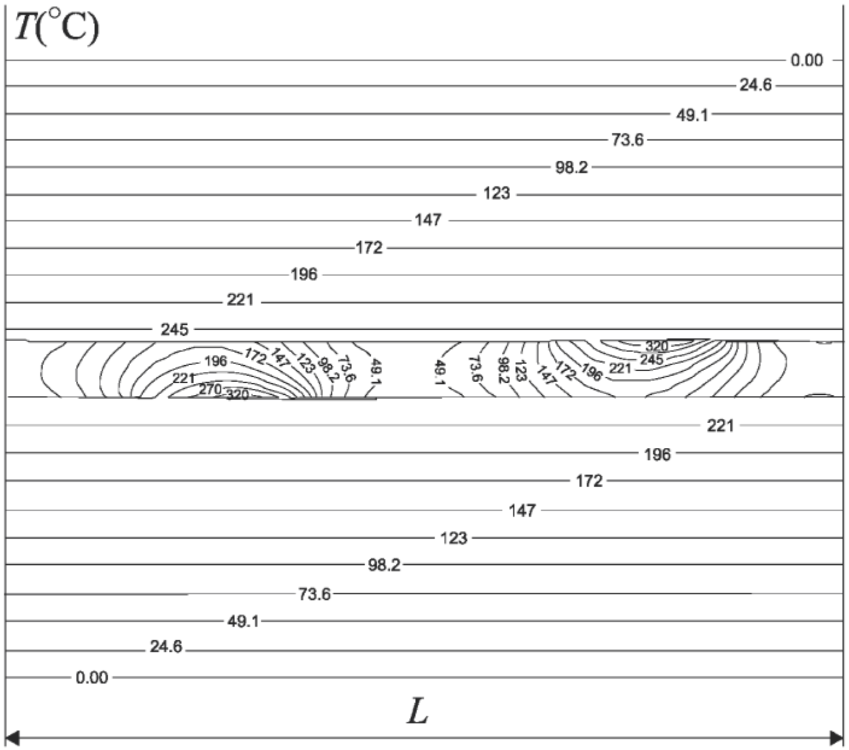
Contact Area above Critical Speed

- Steady state contact pressure between the layers for a dimensionless wavenumber ma=0.334 and sliding speed $V=1.5V_{cr}$
- At this sliding speed, about 23% of the interface is in contact and the maximum contact pressure is about seven times the applied mean pressure.



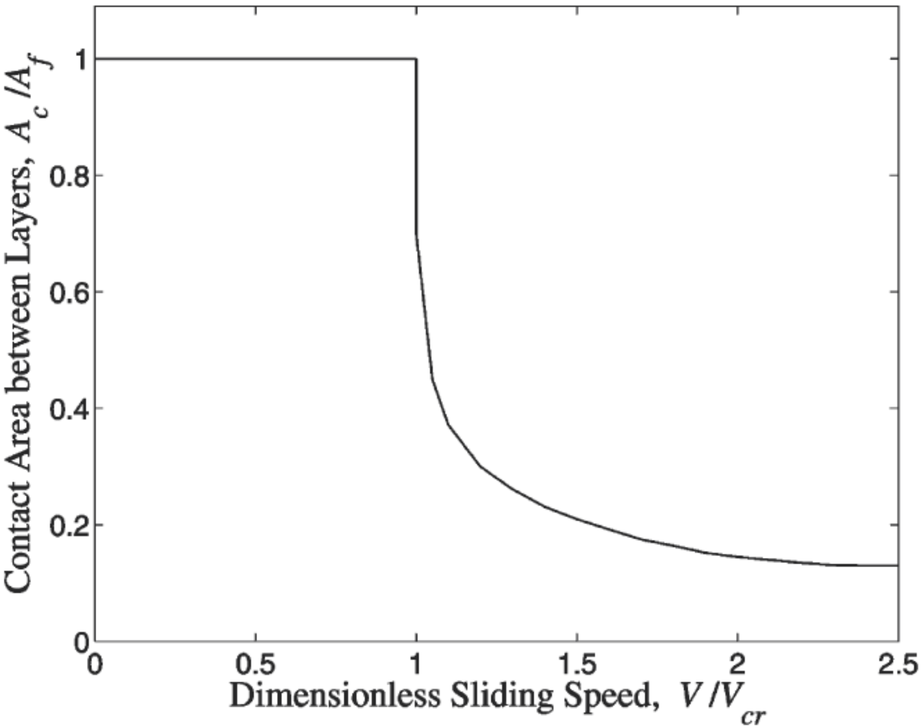
Temperature Contour at Steady State Showing the Position of Hot Spots

- Notice how the temperature fields in the two friction material layers Ω_1, Ω_3 are almost one-dimensional.
- This occurs because the perturbation is almost stationary in the steel layer (the better conductor) and hence corresponds to a very high Peclet number ($>10^4$) in the friction material layers.
- As a result, the deviation from the mean one-dimensional thermal conduction state is restricted to a very thin boundary layer adjacent to the interface.



The Steady-State Contact Area Ratio as a Function of Sliding Speed for Wavenumber ma=0.334

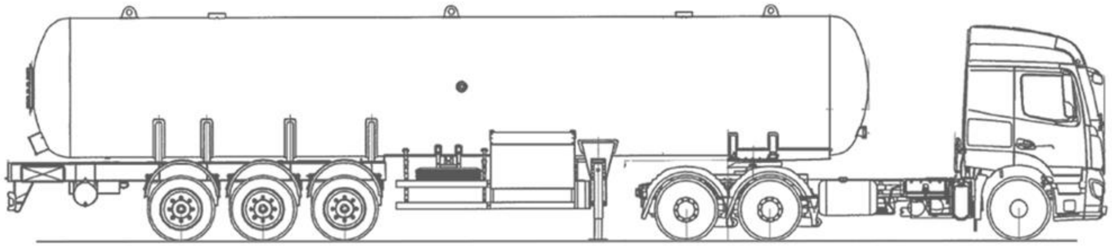
- Shows the effect of sliding speed on the extent of the contact area A_c.
- Below the critical speed, full contact area, represented in the dimensionless form $A_c = A_f = 1$
- Above the critical speed, there is a rapid decrease in the extent of the contact area, which becomes a slower decrease at higher speeds.



4.1.11 Thermoelastic Sliding Bodies Conclusion

- The model is verified with thermotical solution
- The difference in results between simulation model and thermotical model is very small
- results are accepted
- Boundary conditions, Model limitation, and assumptions are accepted.

4.2 Petroleum Gas (LPG) Tanks



4.2.1 Background

- Stability of the LPG trailers depends on
  - the amount of the liquid present in the tanks.
  - a sudden stop/start of the truck with “a speed”
  - taking turn with “a speed”
- The above situations generate “slosh” in the Liquid Petroleum.
- The driver skills and the road/highway quality is also important.
- The location of the “Center of Gravity” in the tank changes with the “slosh” which may affect the stability.
- In tank trucks, e.g., the principal vessel accelerations which excite slosh motions of the fluid are developed during acceleration cornering, maneuvering and braking.

- The slosh "problem" arises when the fluid respond to these accelerations imposing such large forces upon the tank, hence the vehicle motions are:
  - seriously disturbed, or even destabilized.
  - force reactions on the tank tend to overstress or fatigue the vessel.
  - Sloshing can be reduced by introducing baffles inside the tank for damping out high frequency fluid motion.
- Therefore, The optimum capacity of the LPG semitrailers can be determined by investigating:
  - The amount of LPG present in the Vessel which causes “slosh” in some critical scenarios
  - “Stresses” in the Vessel caused by the LPG pressure.

4.2.2 Model Analysis

- In order to consider the complexity of the problem, and to simulate real engineering cases, a numerical simulation using hydrodynamic equations for the fluid, and structural dynamic equations for the tank with appropriate material laws need to be used.
- Arbitrary Lagrangian Eulerian (ALE) finite element methods technique is used to solve fluid sloshing tank problems and fluid force loading on the structure for the capability to control mesh geometry independently from material geometry.
- Fluid-structure interaction problems are strongly dominated by the coupling of the induced fluid oscillations and the resulting motion of the structure.

- <u>Sloshing in the longitudinal direction</u>
  - Semitrailer is moving with a speed of 80 km/h on a straight road
  - The standard distance traveled by the trailer to come to a complete rest is 35m.

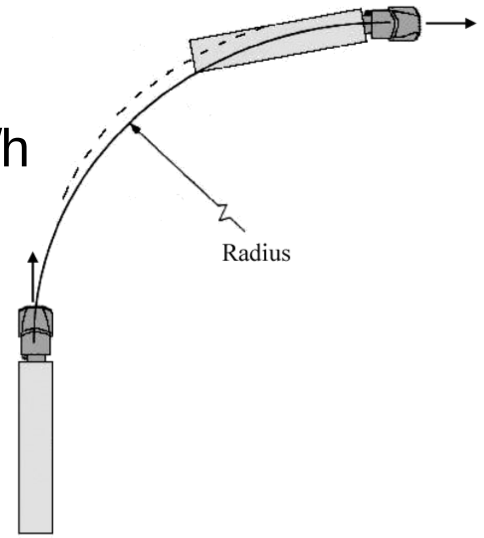
$$a_{long} = \frac{v_2^2 - v_1^2}{2S} = 7.1\ \mathrm{m/s^2}$$

$$t_{long} = \frac{v_2 - v_1}{a_{long}} = 3.15\ \mathrm{s}$$

- <u>Sloshing in the lateral direction</u>
  - truck is taking a steady U-Turn with a speed of 40 km/h
  - The turn radius is 30m

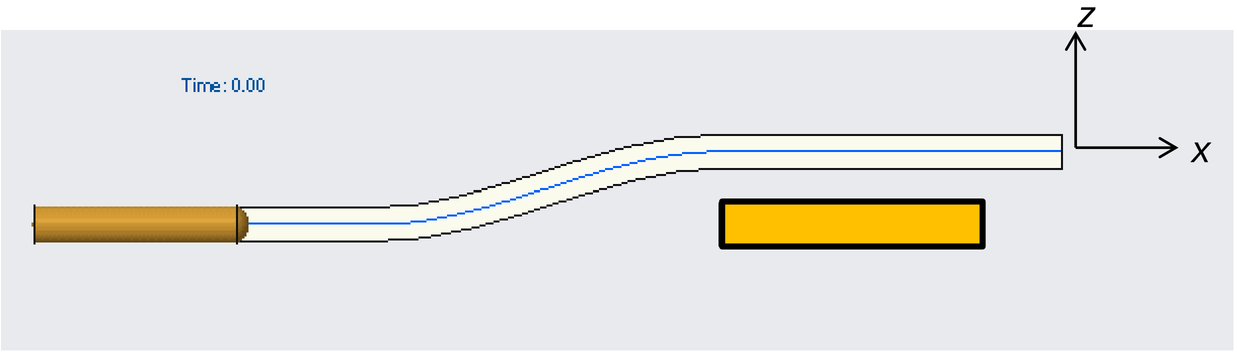
$$a_{late} = \frac{v^2}{r} = 4.1\ \mathrm{m/s^2}$$

$$t_{late} = \frac{\pi r}{2v} = 8.5\ s$$

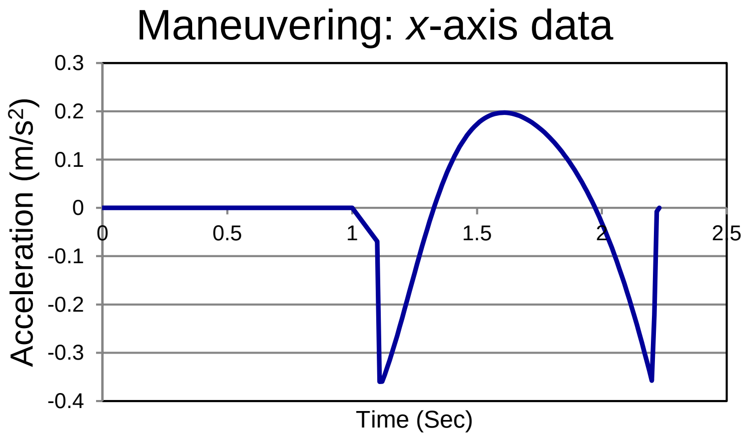


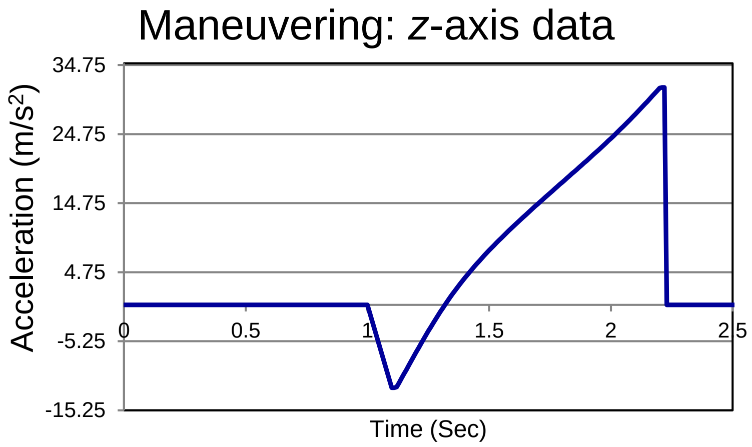
Sloshing during a Maneuver

- The truck is overtaking another vehicle at a speed of 65 km/h
- The tank acceleration is determined as shown in the animation
- The acceleration data is used to simulate the LPG slosh

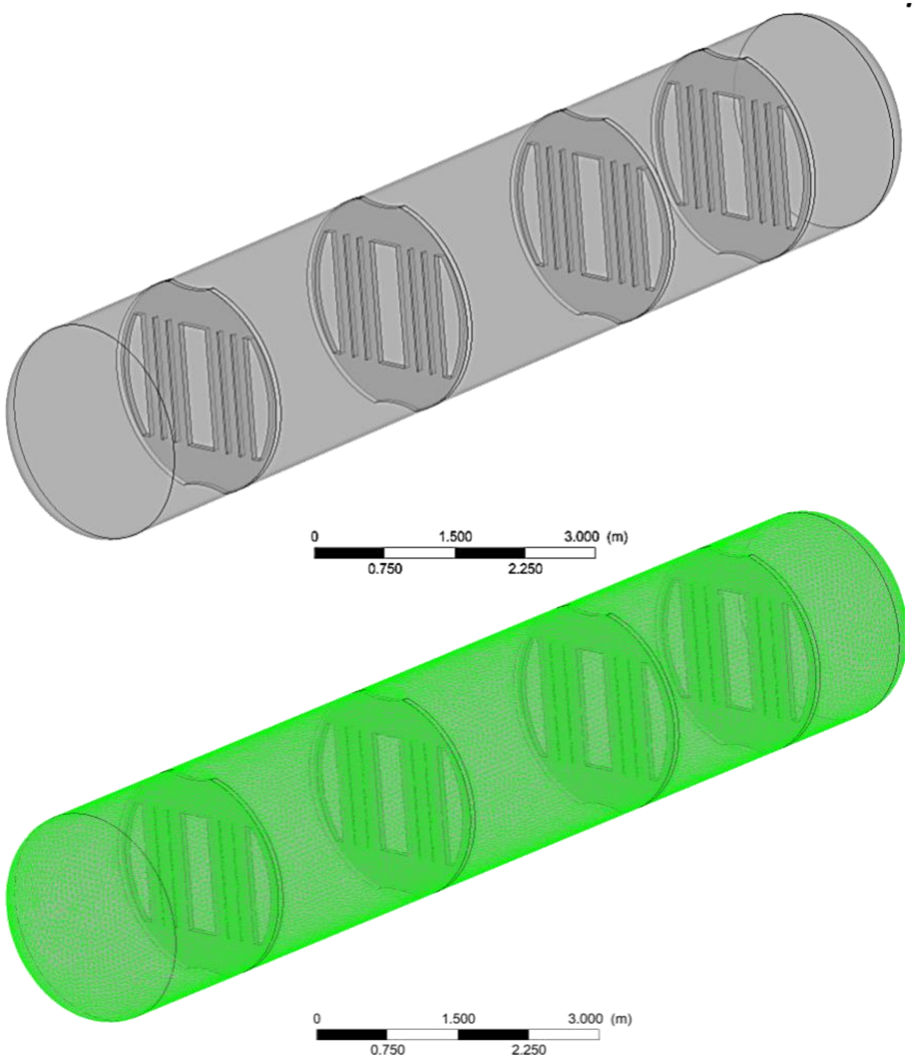


- The acceleration data is generated using Creo Mechanism®





- 2.6 million tetrahedra elements
- Homogenous, Multiphase Flow model
- *K-E* turbulence model
- Transient simulation, 10s Longitudinal, 15s Lateral and 15s Maneuvering.
- 0.01 time step to achieve the convergence.



4.2.3 LPG Center of Gravity

$$x_{cg} = \frac{1}{\bar{V}} \sum_{i}^{LPG} x_i V_i \rightarrow (4a)$$

$$y_{cg} = \frac{1}{\bar{V}} \sum_{i}^{LPG} y_i V_i \rightarrow (4b)$$

$$z_{cg} = \frac{1}{\bar{V}} \sum_{i}^{LPG} z_i V_i \rightarrow (4c)$$

- $\bar{V}$ is the total volume of the LPG in the tank
- x_i, y_i and z_i are the coordinates of the center of the cell *i*
- V_i is the volume of the cell *i*

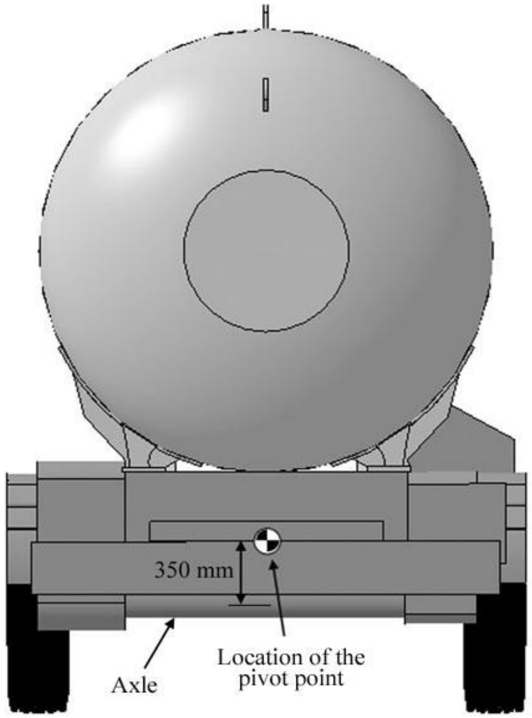
The Vessel tilt simulation model

- Assumptions
  - The weights of the Vessel, chassis and others are the same in all the cases and are therefore ignored.
  - The spring stiffness and its damping is also ignored.
  - The weight of the LPG is directly proportion to the angular displacement (tilt) of the Vessel
  - The Vessel tilts about a pivot point located at 350mm above the center of the axle.
  - The 90% filled Vessel tilts 5^{o} maximum.

Filling	**97%**	**90%**	**80%**	**70%**	**50%**
Weight (kN)	268.5	256	232	202	132
Max. tilt (deg)	5.4	5	4.53	3.95	2.6

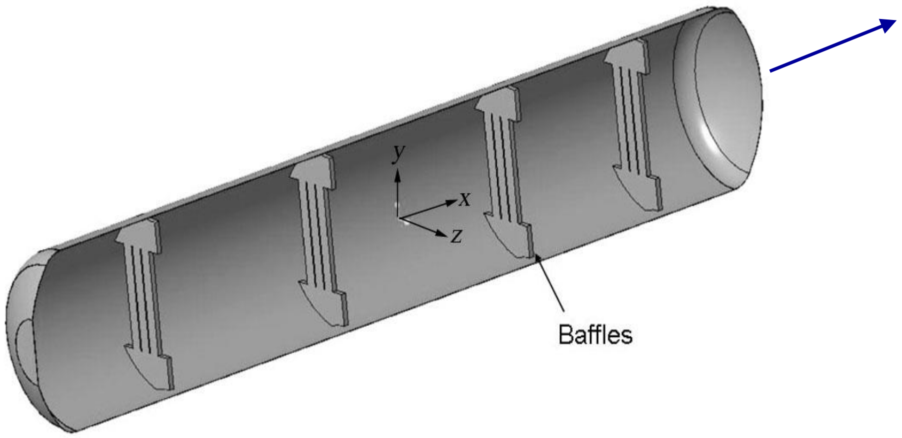
4.2.4 The Tank tilt simulation model…

- The Tires and axles are modeled as 1st part
- The Tank and chassis is modeled as 2nd part.
- The 1st part is fixed whereas the 2nd part is pin-connected at the center of the axle as shown.

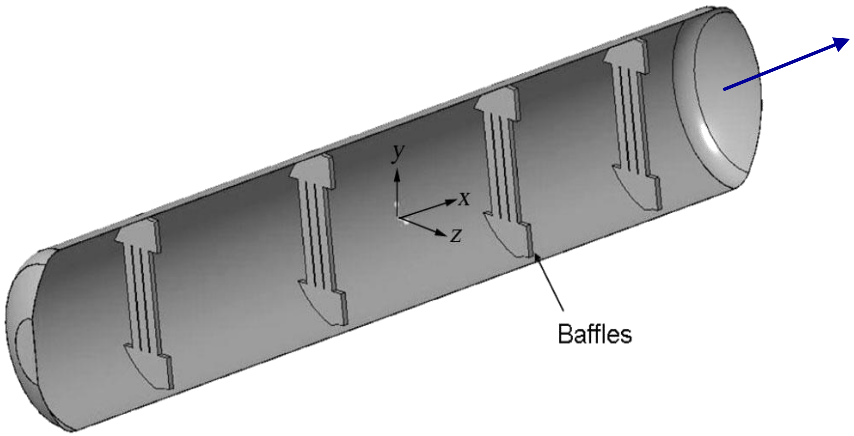


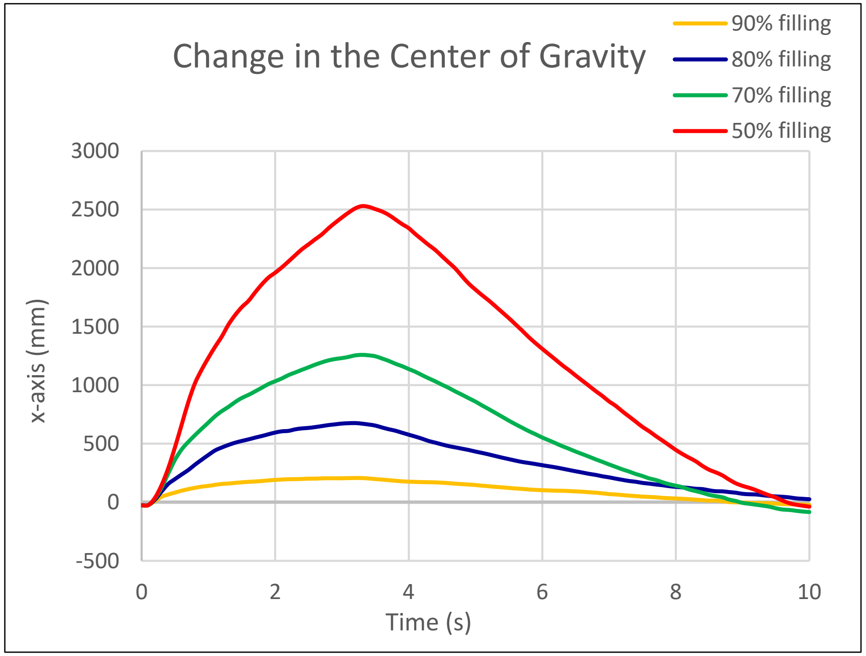
Case 1:
Applying Emergency Brake on a straight road

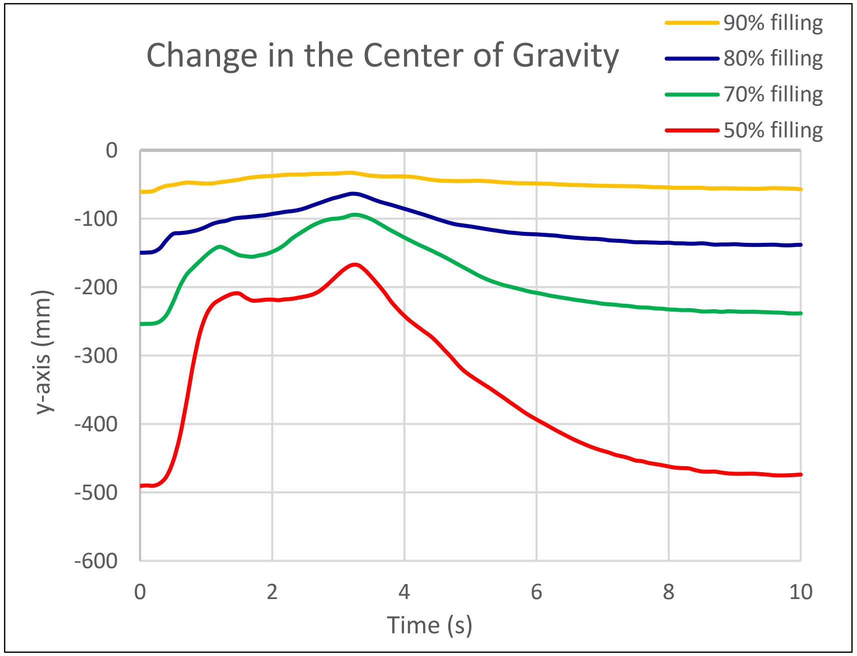
- Assumption
  - Speed = 80 km/h
  - The distance traveled by the trailer to come to a complete rest = 35m.
  - Simulation time = 10s



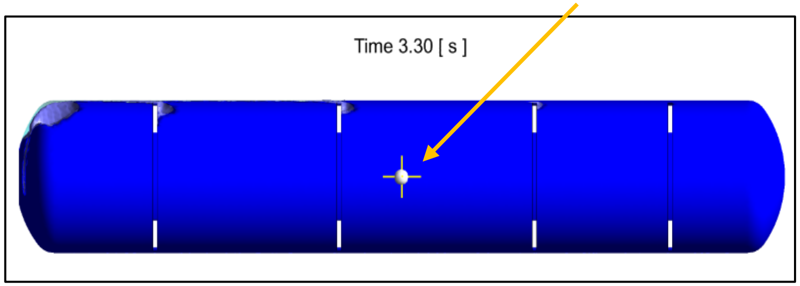
- Change in CG in the longitudinal direction (x-axis)



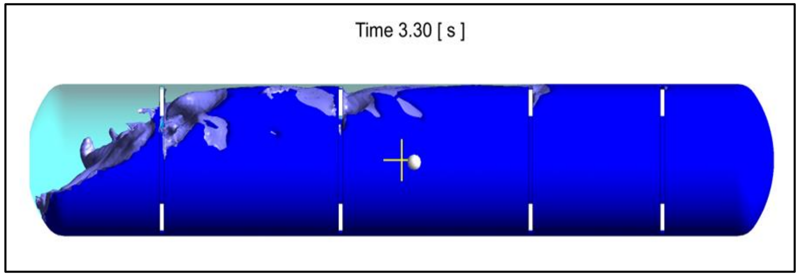




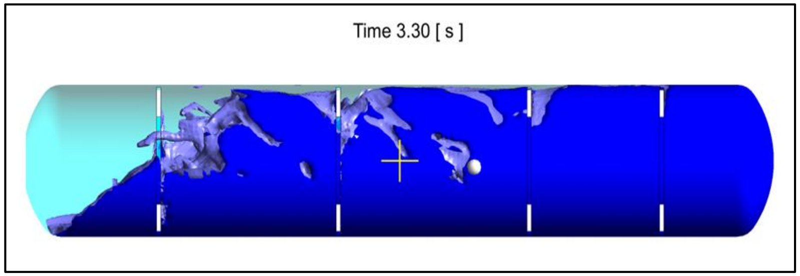
- White dot represents CG, + is the coordinate center.



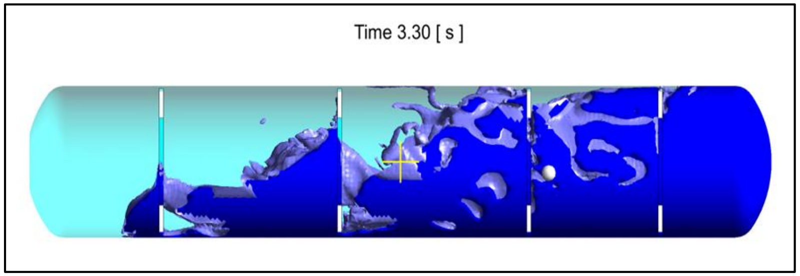
97%



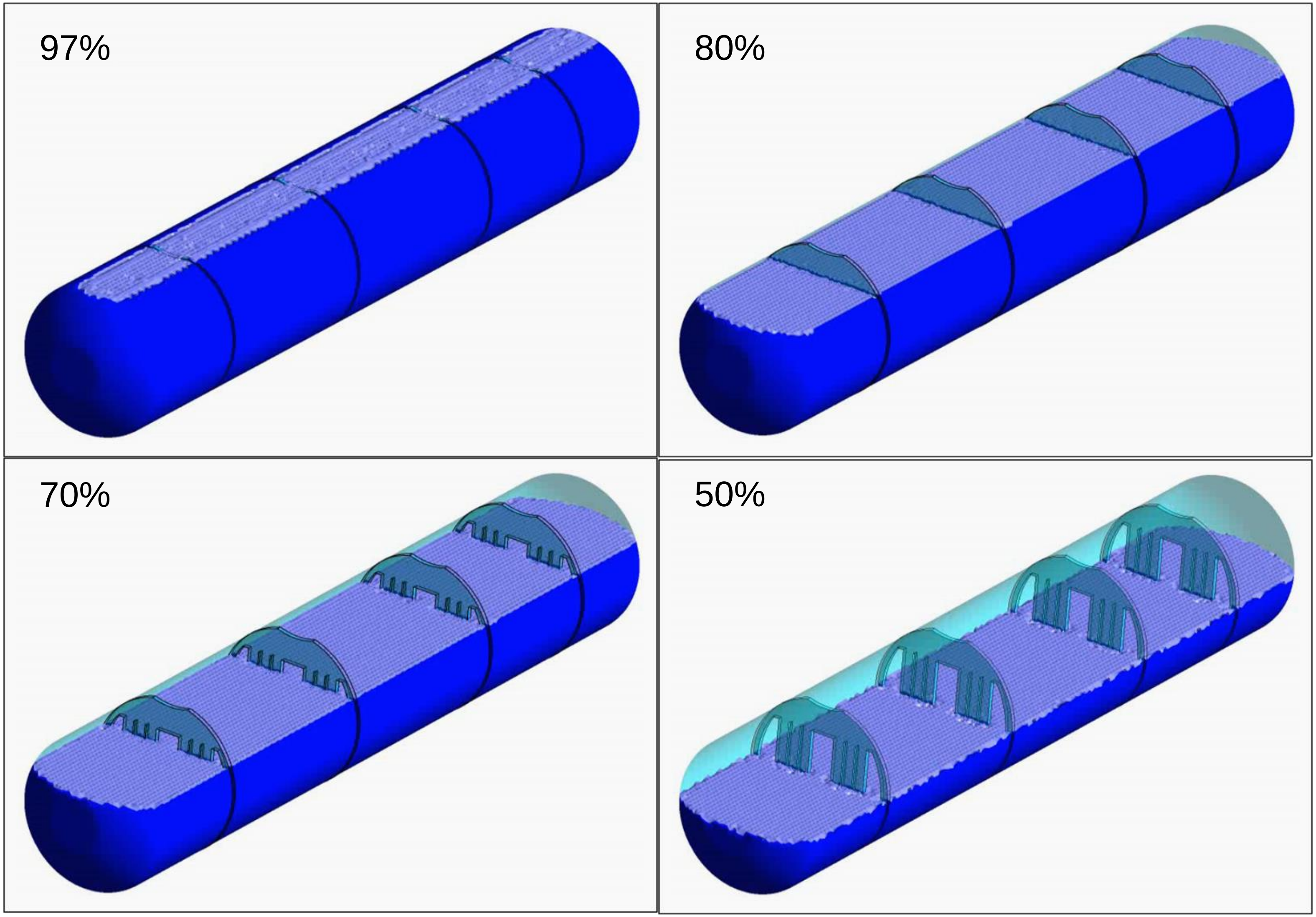
80%



70%

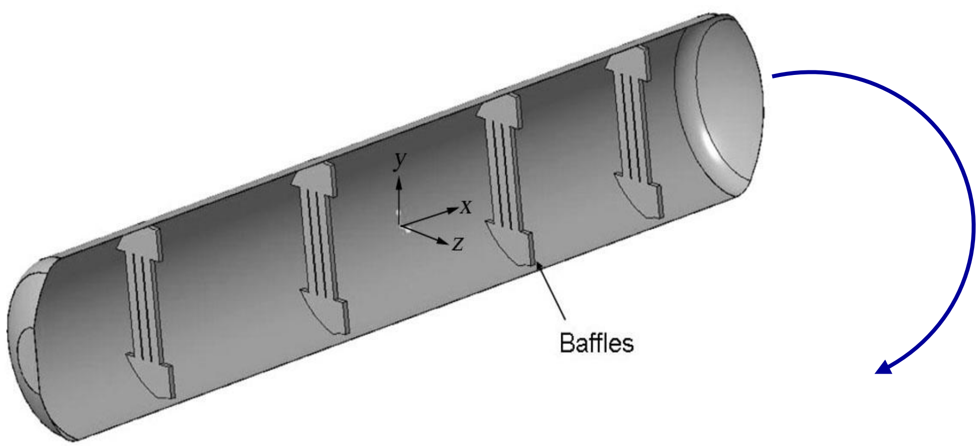


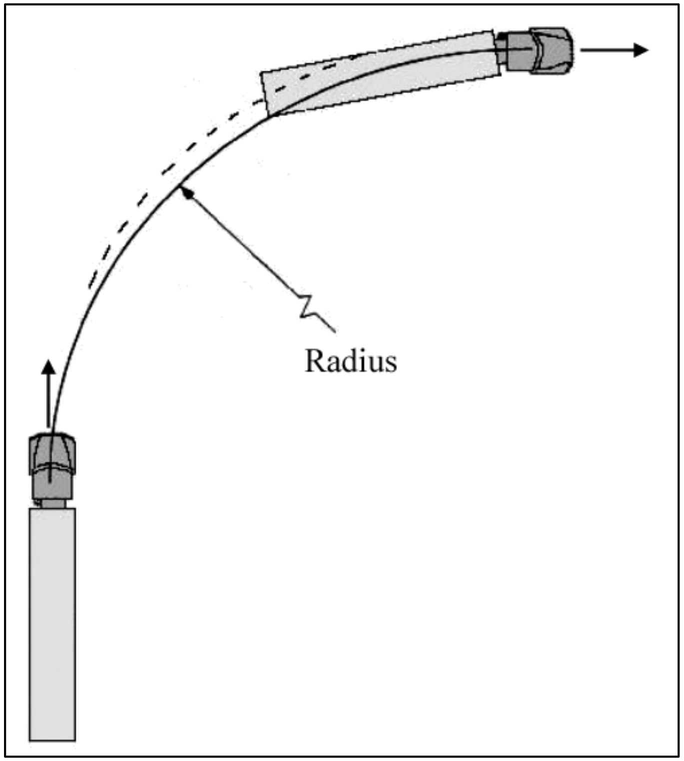
50%


97%
80%
70%
50%

Case 2:
Taking a steady U-Turn to the right (*z*-axis)

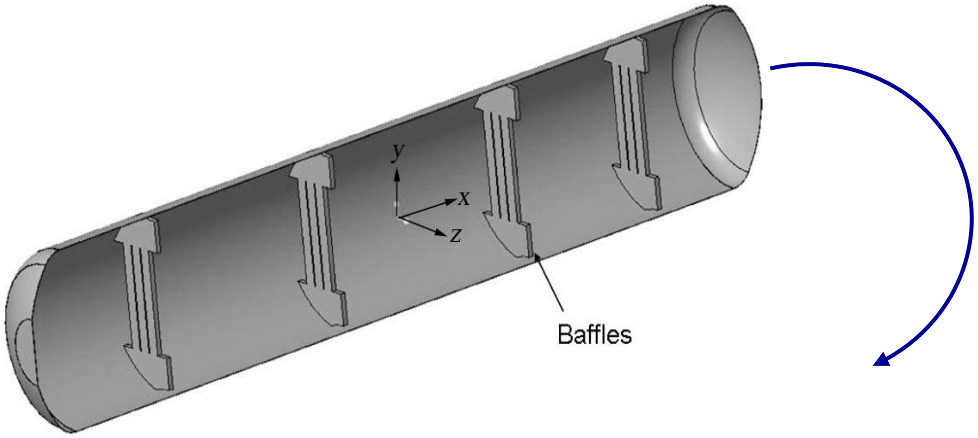
- Assumptions
  - Speed = 40 km/h
  - The turn radius = 30m
  - Simulation time = 15s

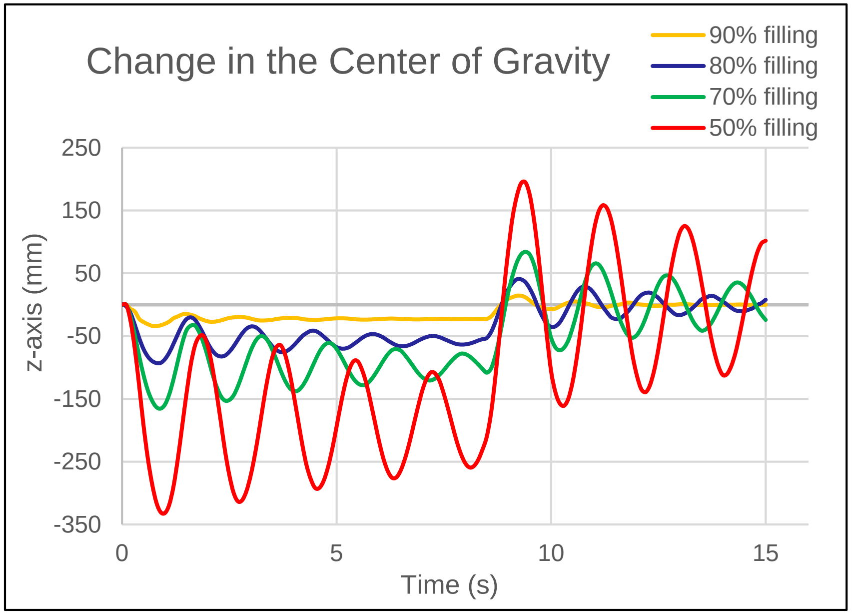


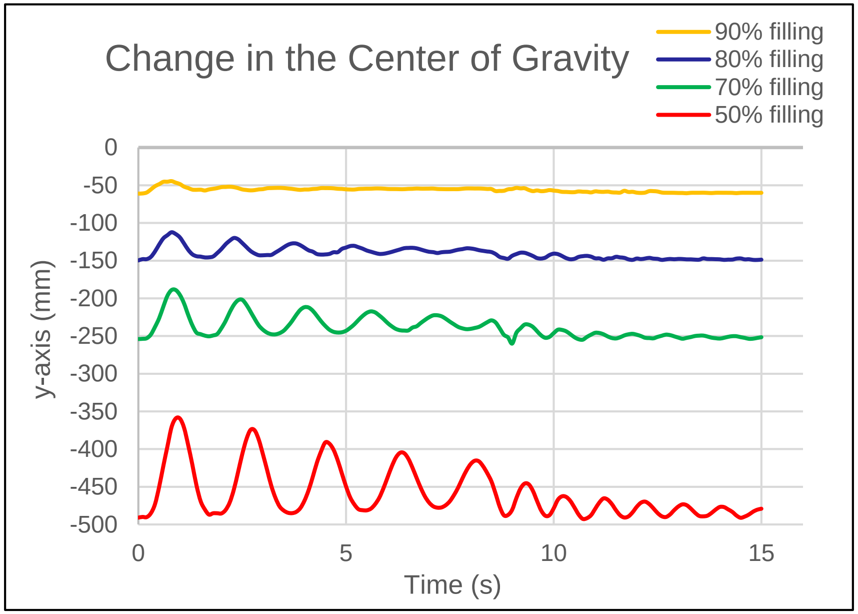


Change in CG in the Lateral direction…

- Change in CG in the lateral direction (z-axis)

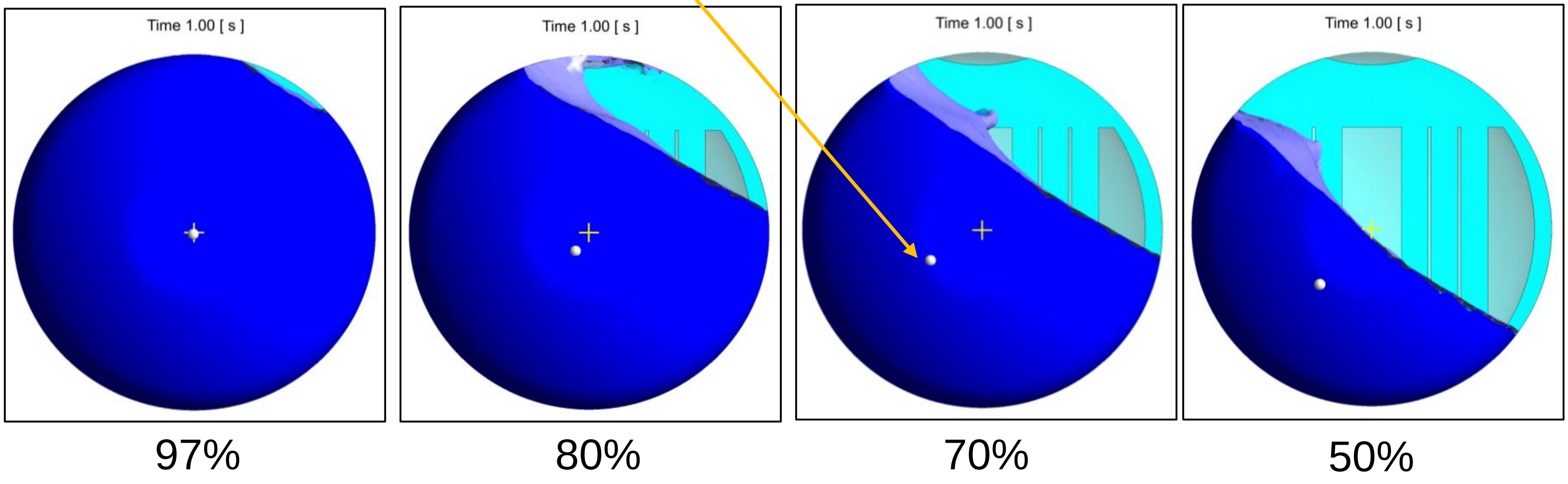


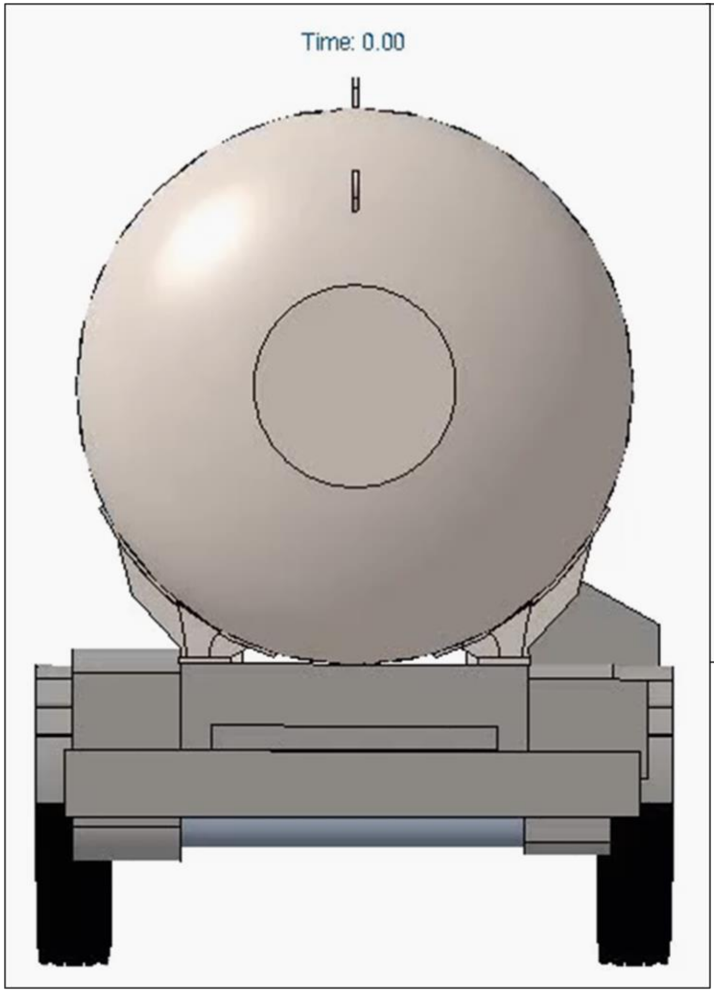


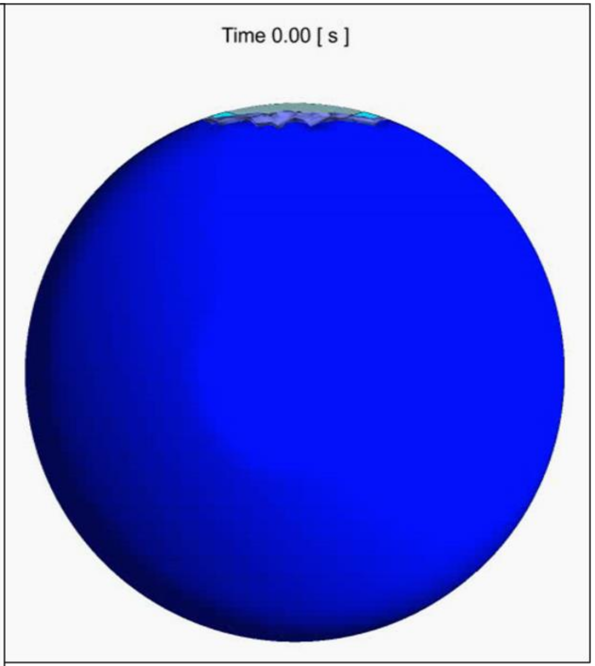


- Change in CG in the lateral direction

  - White dot represents CG, + is the coordinate center.



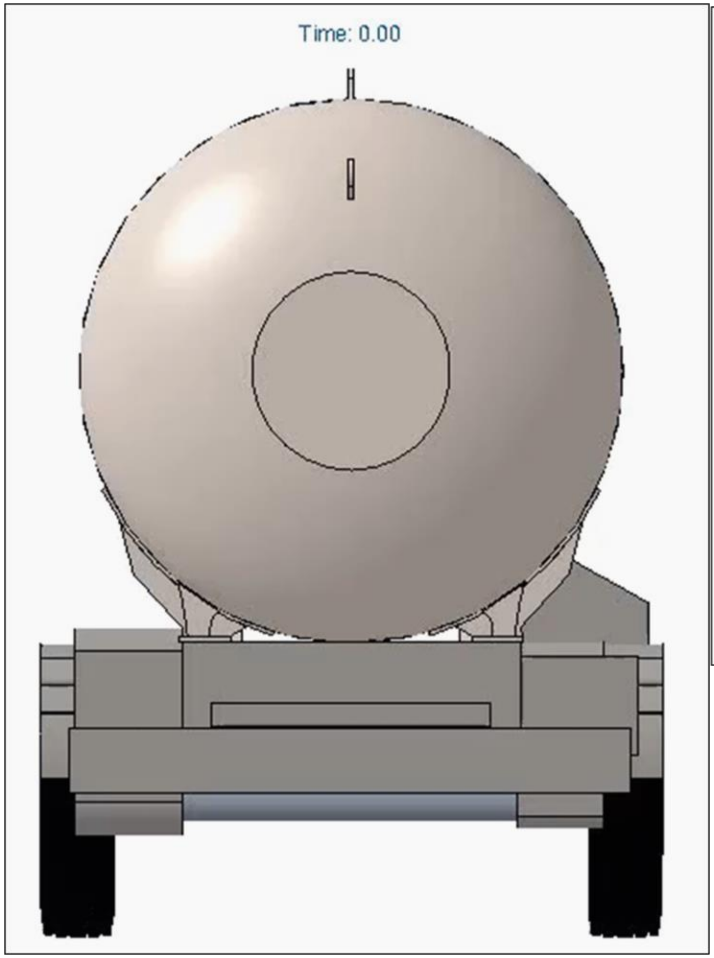


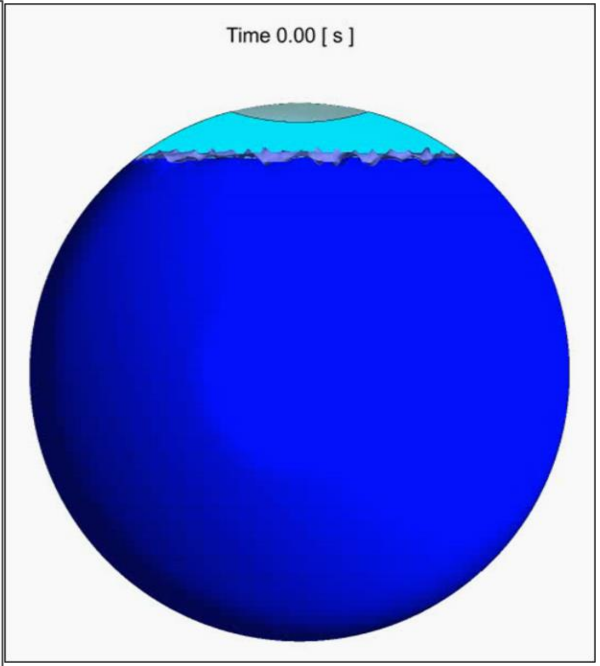


97% filled Vessel, turning right

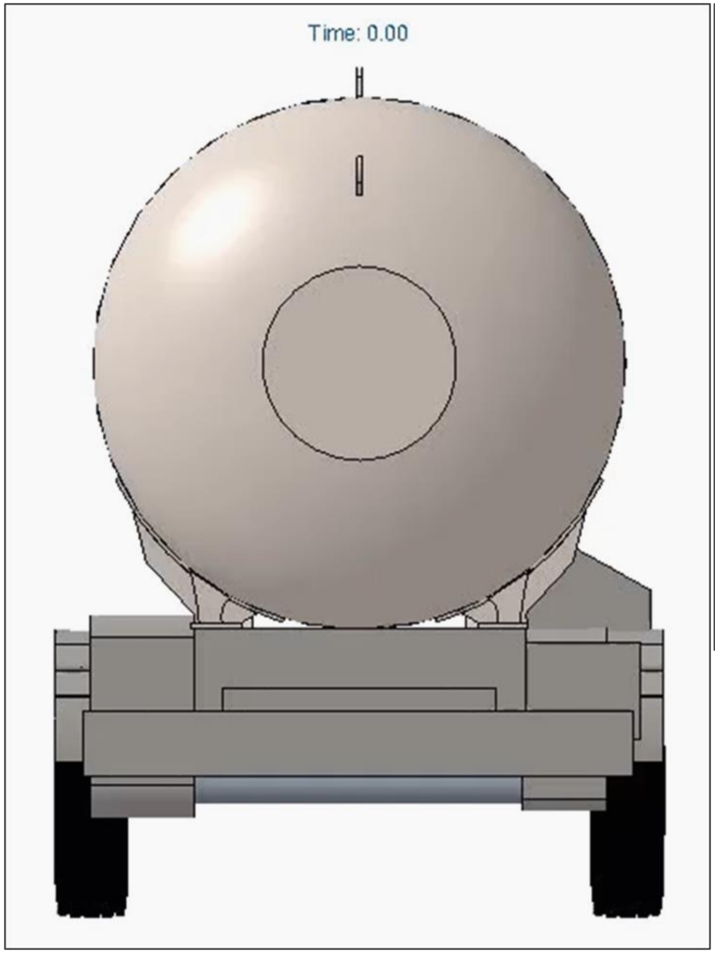
LPG weight = 268.5 kN. Max. tilt = 5.4º

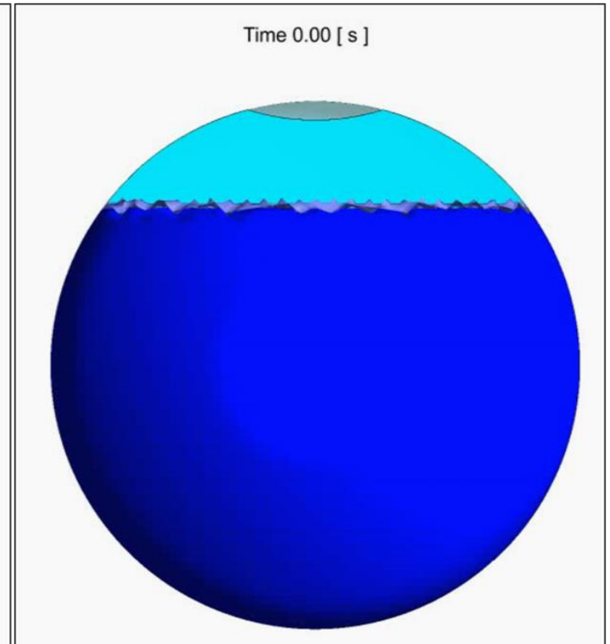
(Back view)



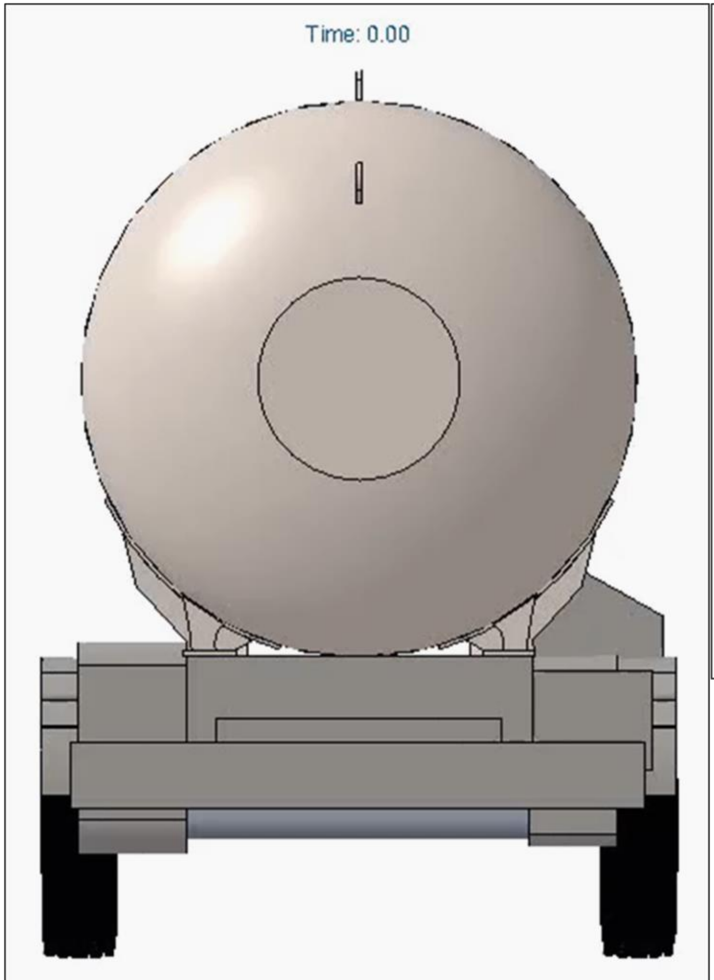


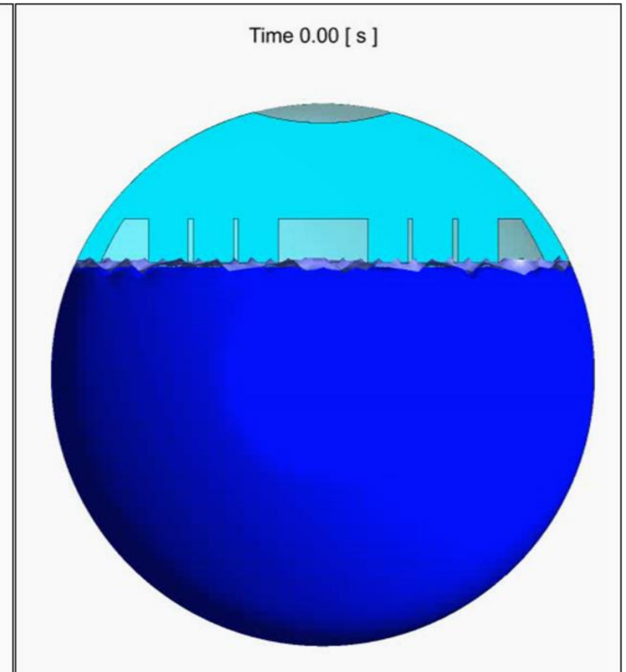
90% filled Vessel, turning right
LPG weight = 256 kN. Max. tilt = 5^o
(Back view)



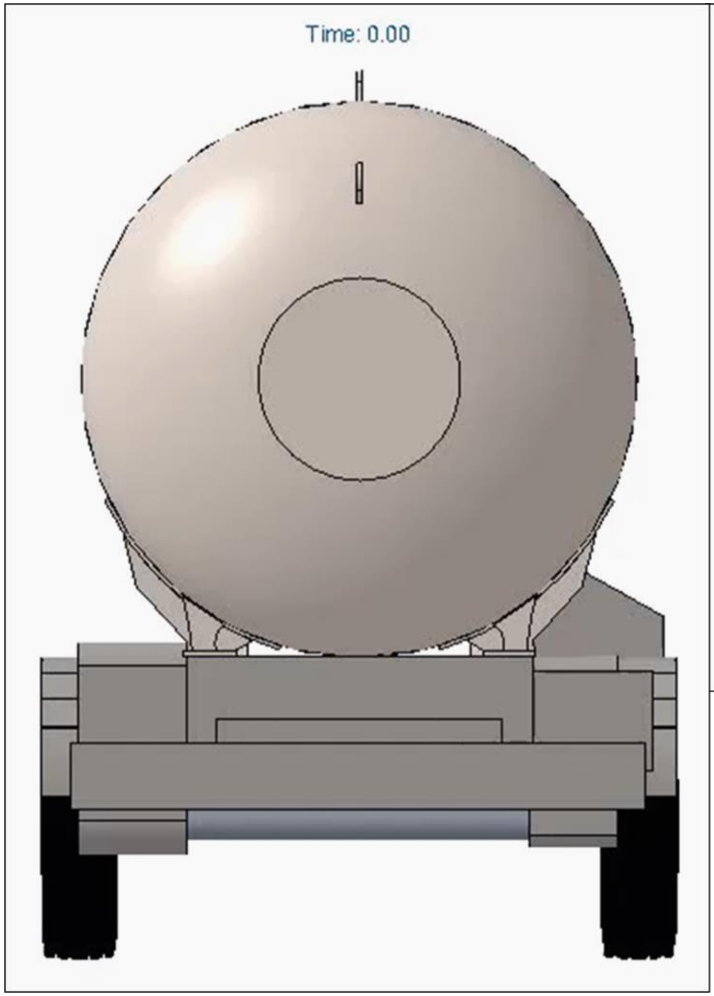


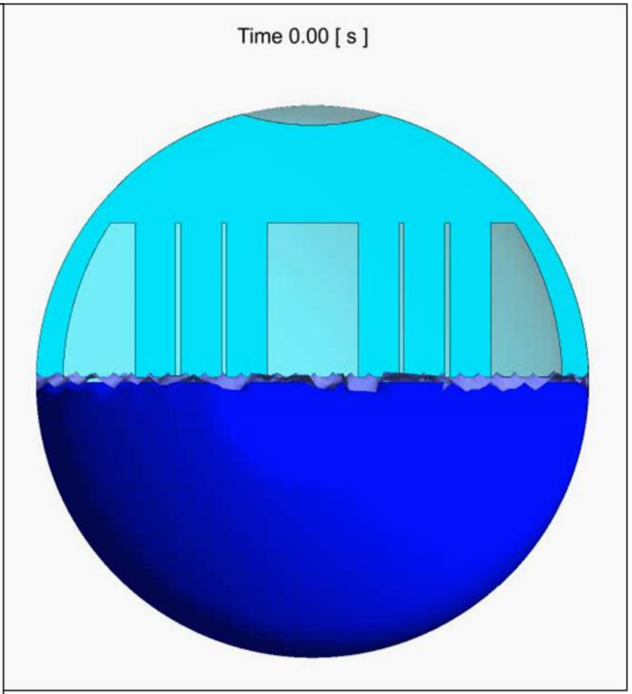
80% filled Vessel, turning right
LPG weight = 232 kN. Max. tilt = 4.53°
(Back view)



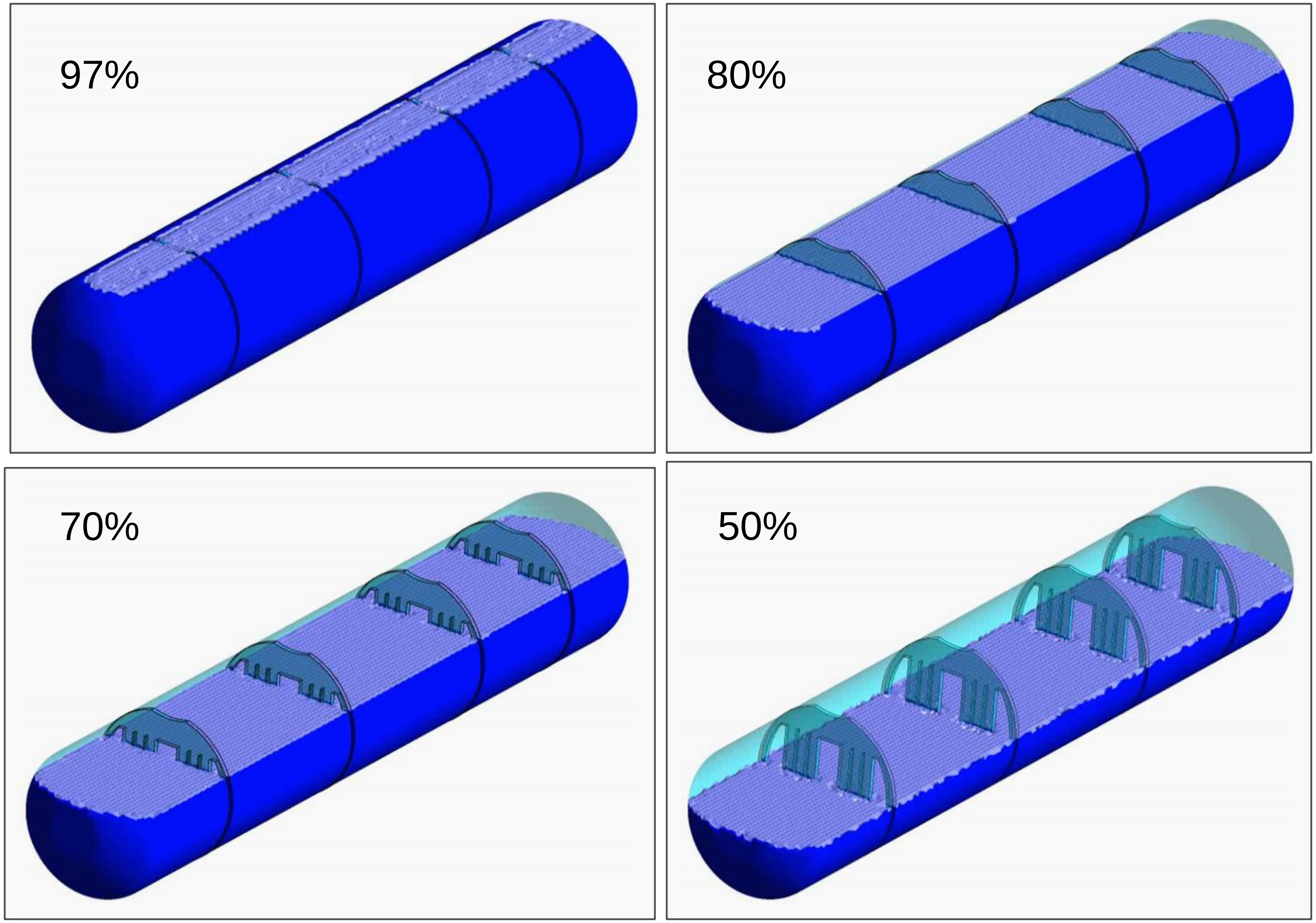


70% filled Vessel, turning right
LPG weight = 202 kN. Max. tilt = 3.95°
(Back view)





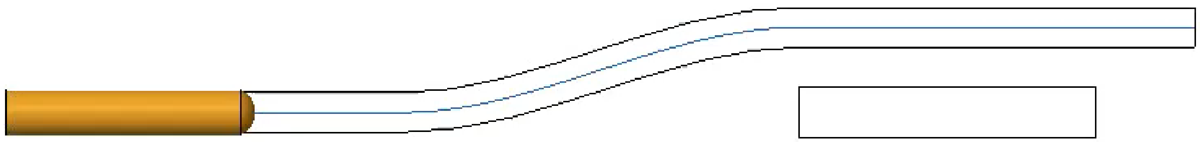
50% filled Vessel, turning right
LPG weight = 132 kN. Max. tilt = 2.6°
(Back view)



Comparison of **slosh** with different fills during a turn to the right

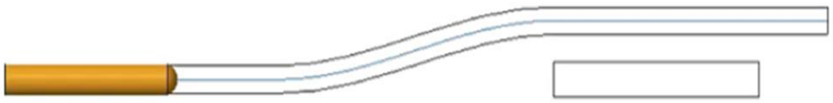
Case 3: Maneuvering

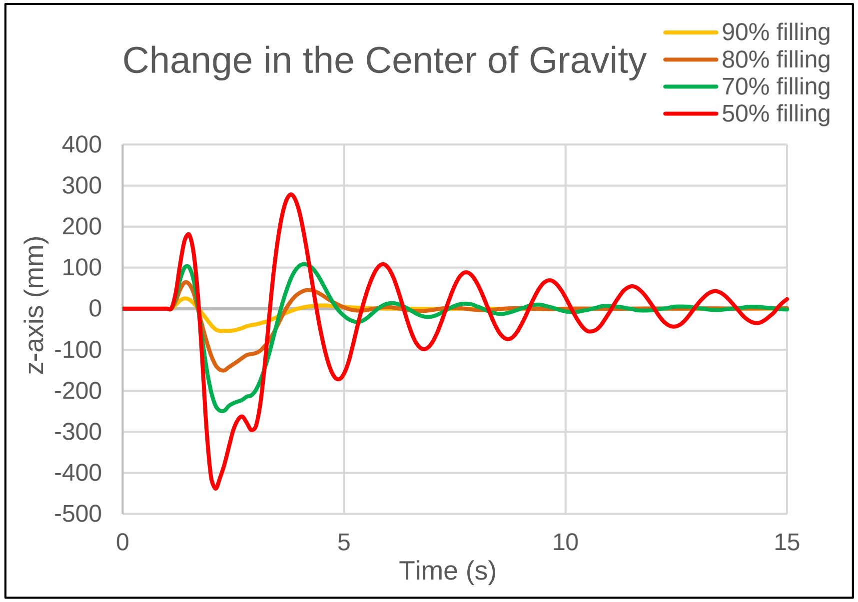
- Assumptions
  - The trailer is overtaking another vehicle at a speed of 65 km/h

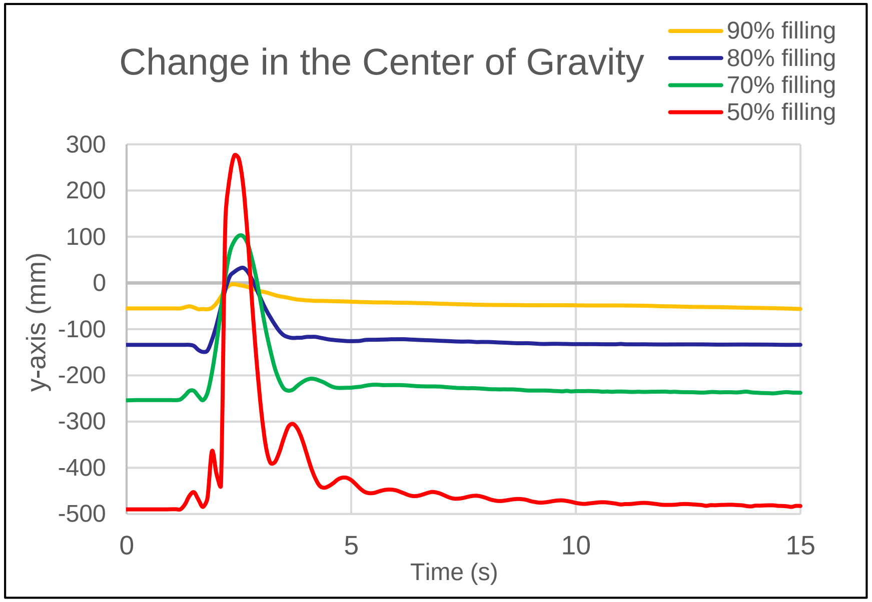


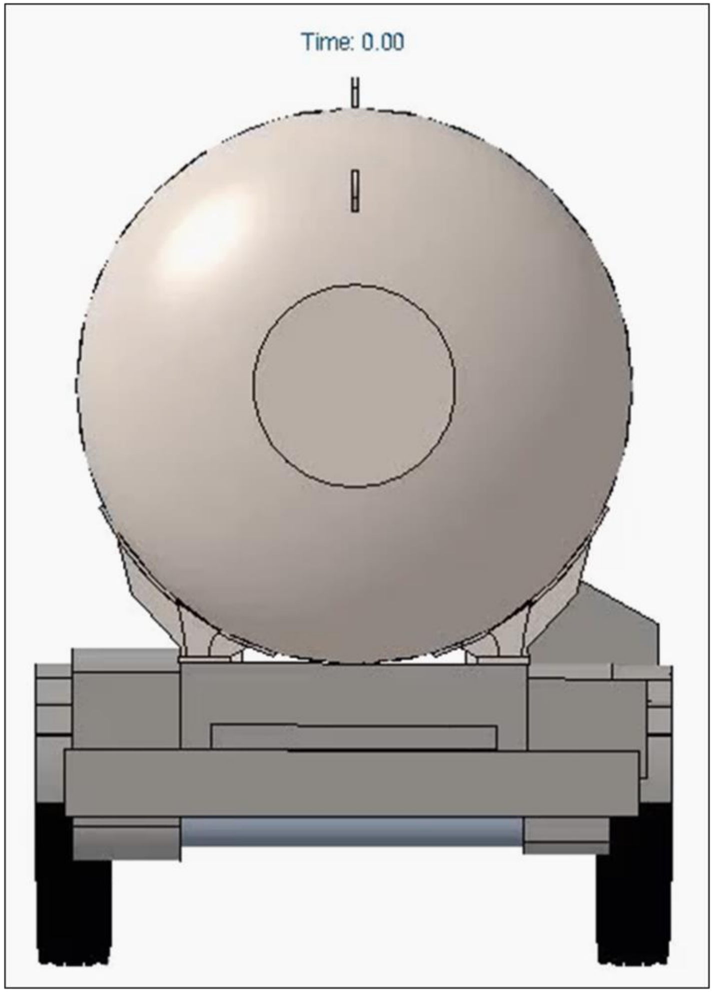
Change in CG during a Maneuver

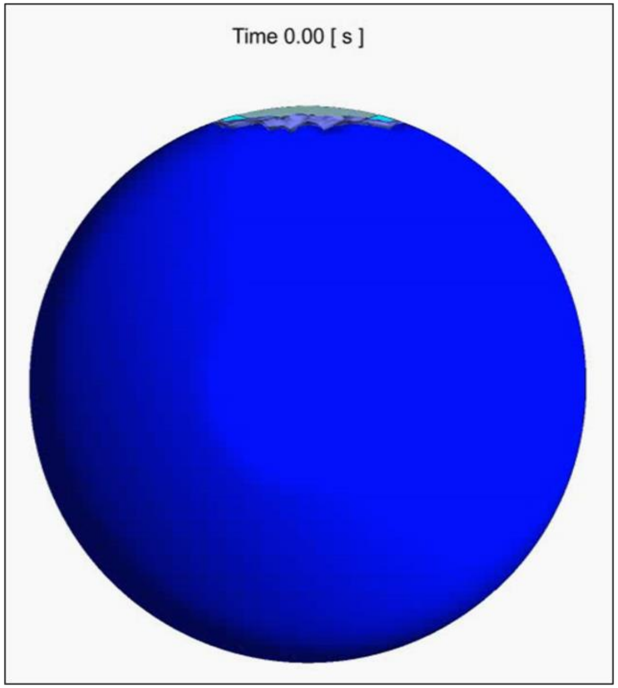
- Change in CG during a Maneuver



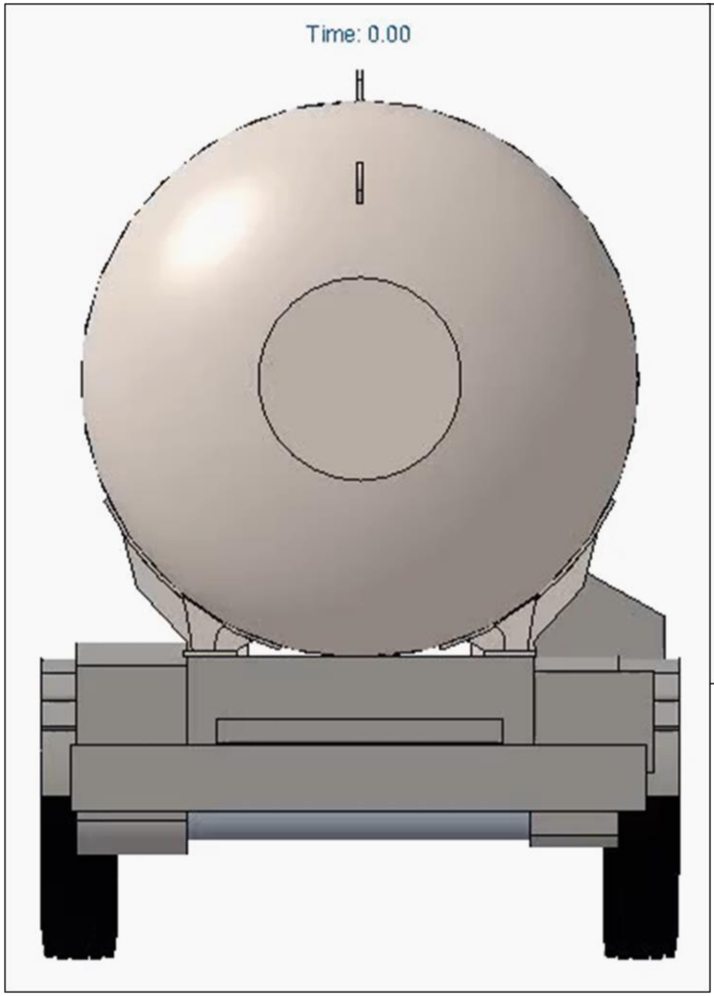


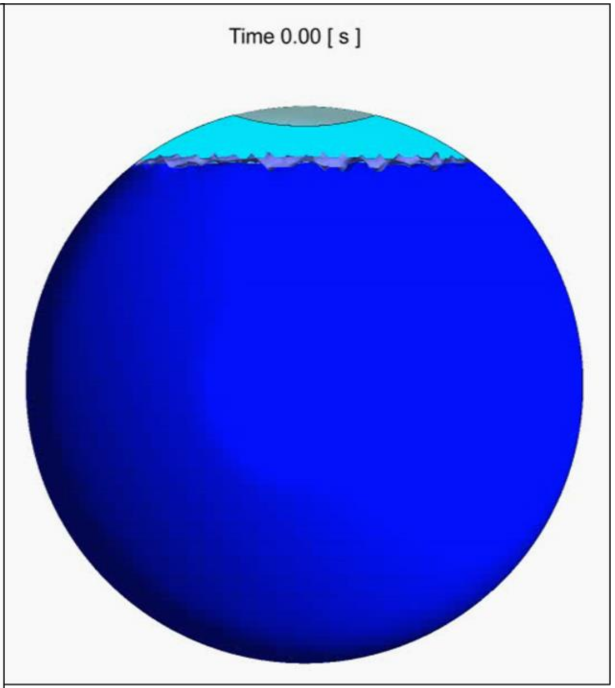




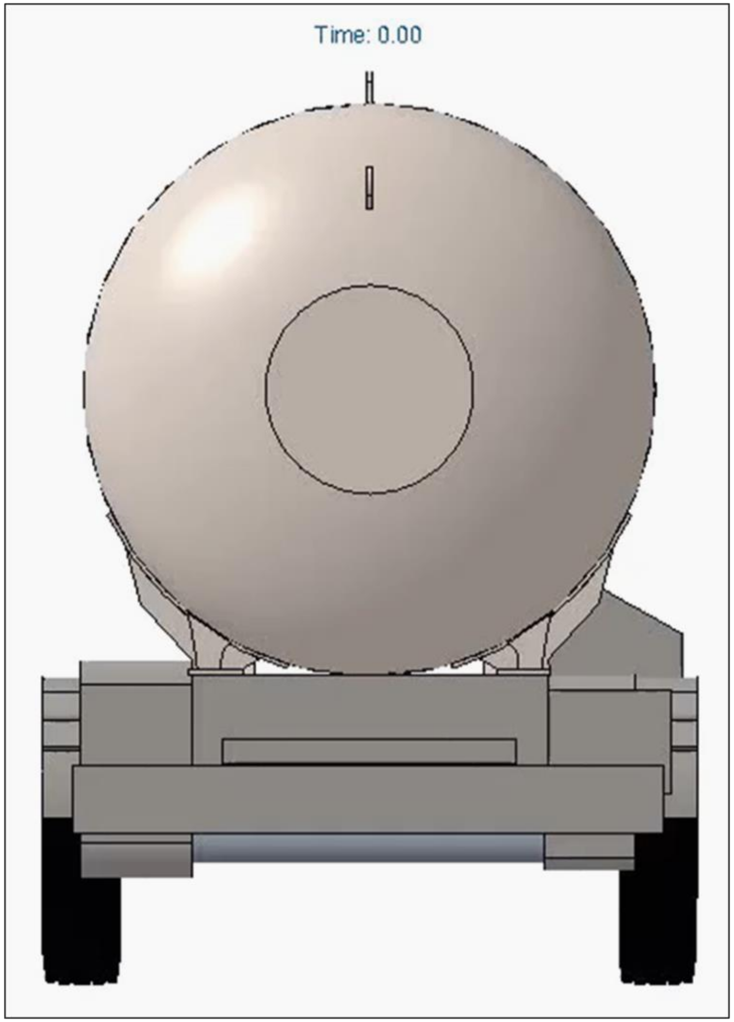


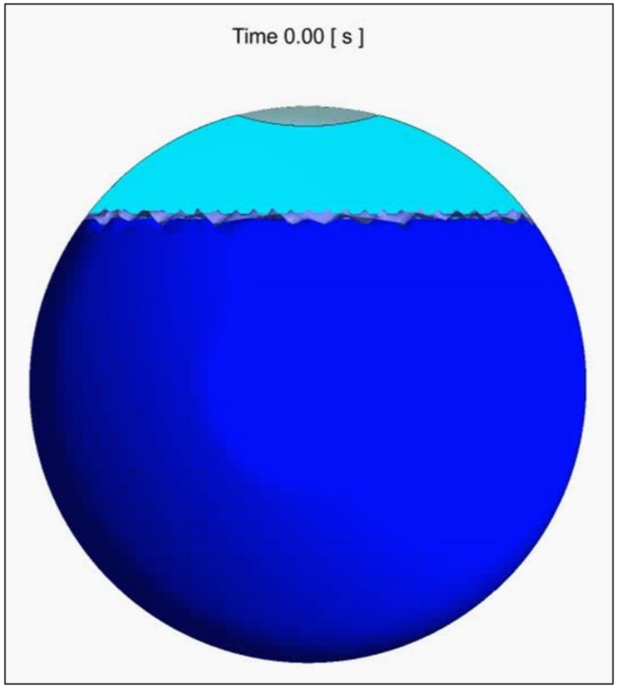
97% filled Vessel, Maneuvering
LPG weight = 268.5 kN. Max. tilt = 5.4º
(Back view)



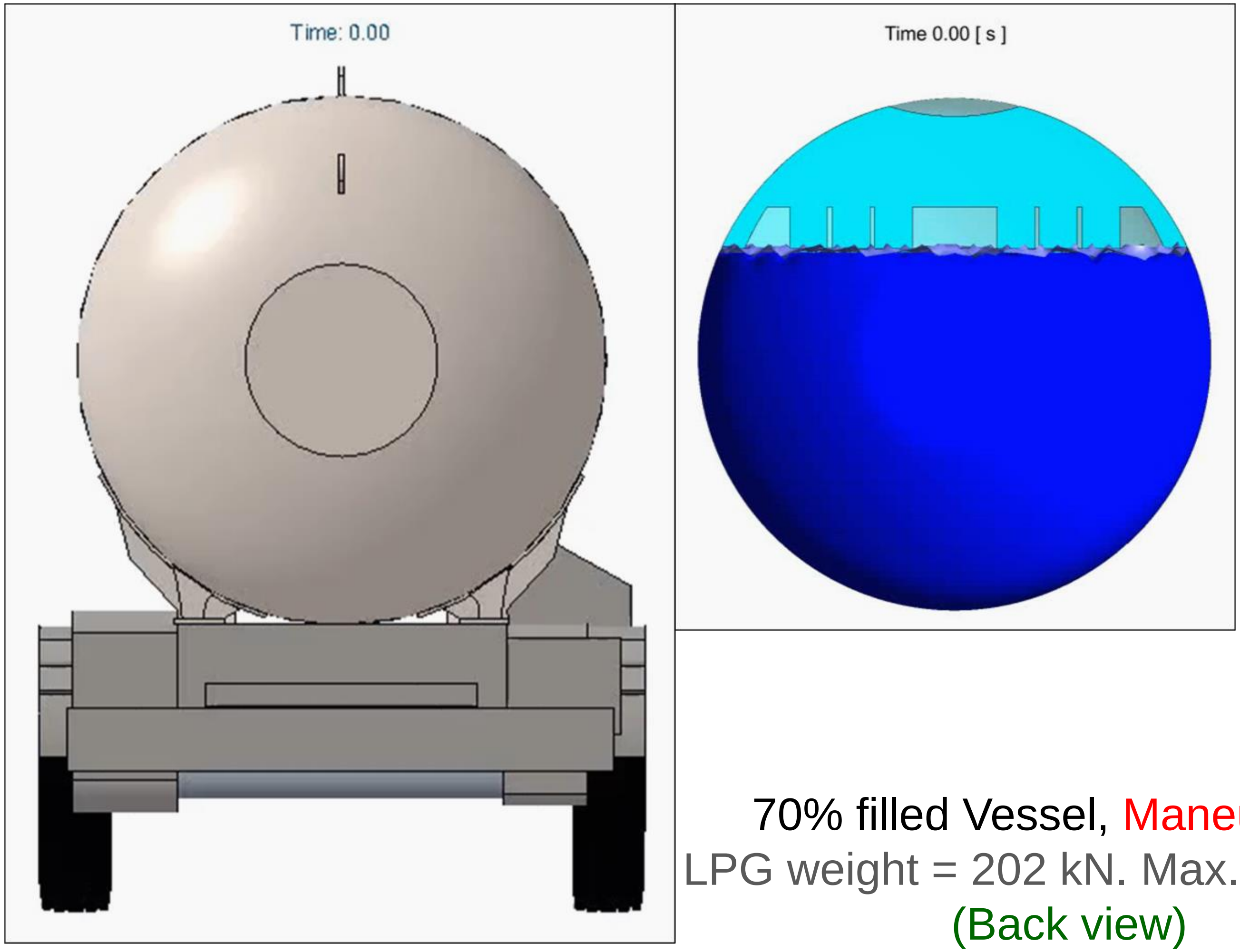


90% filled Vessel, Maneuvering
LPG weight = 256 kN. Max. tilt = 5^o
(Back view)

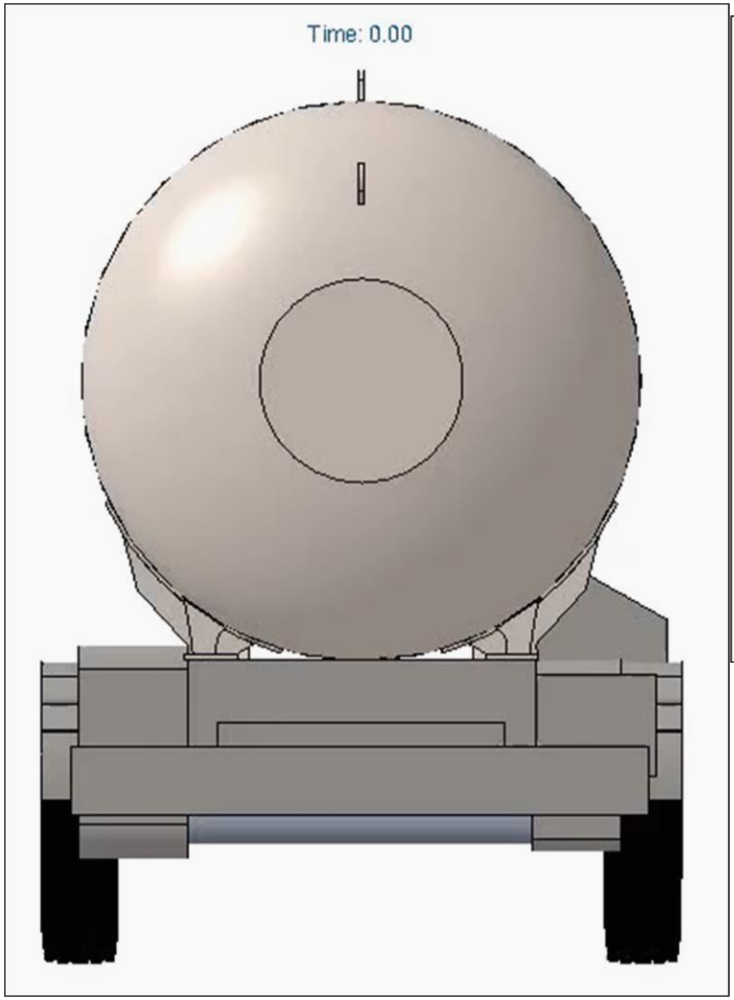


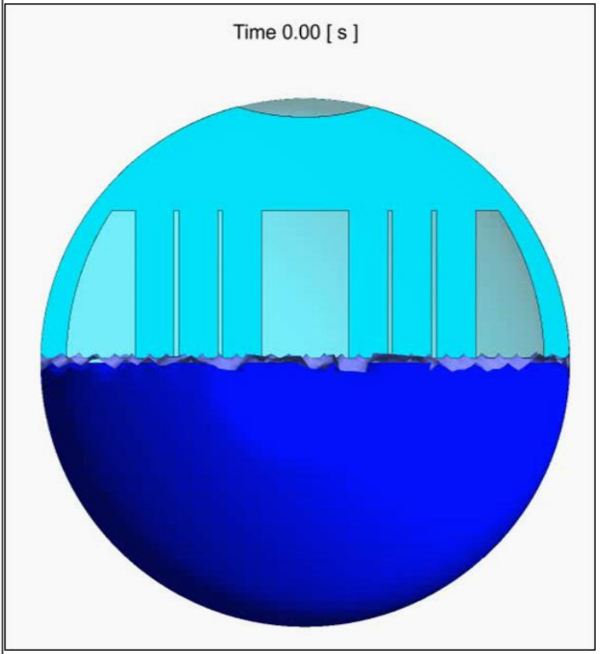


80% filled Vessel, Maneuvering
LPG weight = 232 kN. Max. tilt = 4.53^o
(Back view)

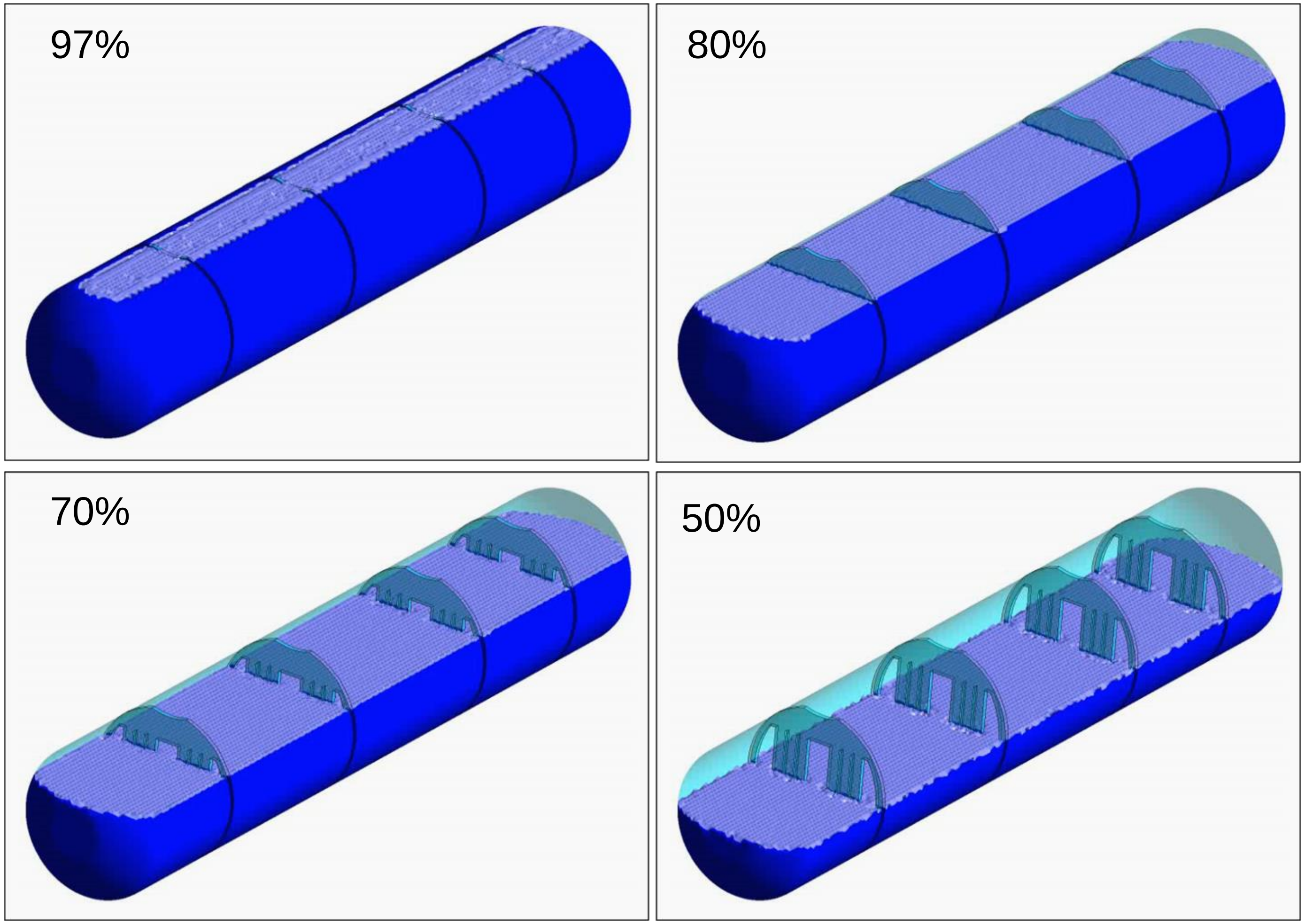


70% filled Vessel, Maneuvering
LPG weight = 202 kN. Max. tilt = 3.95°
(Back view)





50% filled Vessel, Maneuvering
LPG weight = 132 kN. Max. tilt = 2.6°
(Back view)



Comparison of **slosh** with different fills during Maneuvering

4.2.5 Conclusions from Sloshing Analysis

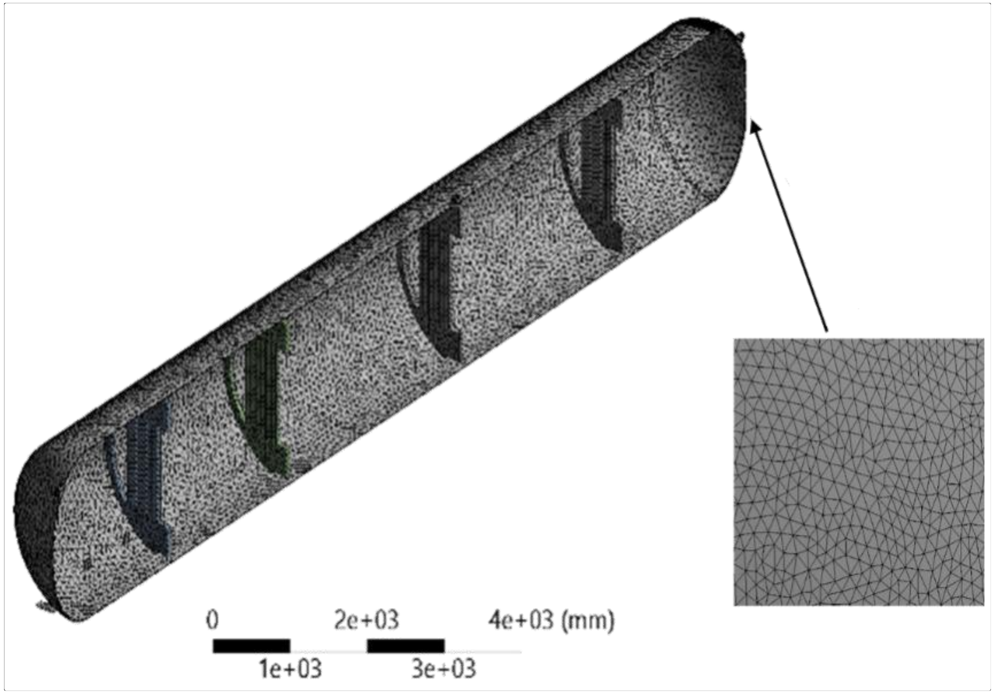
- The tank oscillation damps out quickly with more fill
- The trailer is stable with even with the highest fill
- Although the tank with the lowest fill oscillates more, but the tilt is small because of the low weight
- there is no threat of instability to the semitrailer weather it is partially or fully filled, unless, the driver not follows the speed limits/driving procedures.
- technician

On the other hand, fully filled tank will have more stresses, thus Stress analysis is conducted.

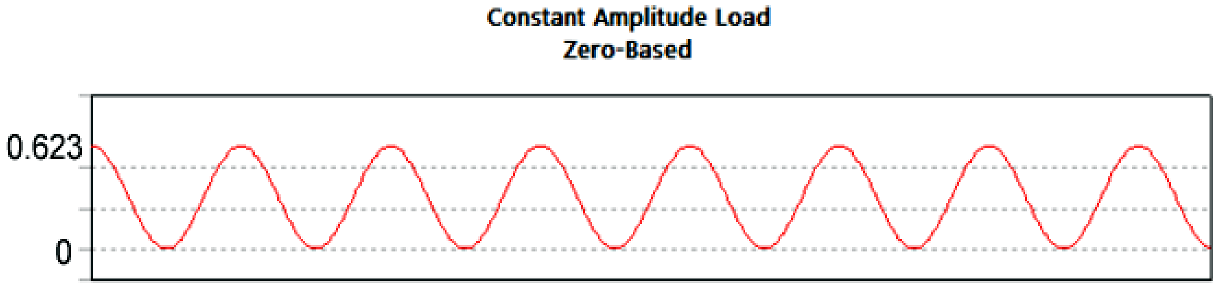
4.2.6 Tank Stress Analysis

- Assumptions
  - Summer conditions are considered.
  - The corresponding tank pressures is 90.4 psi.
  - Two load conditions are analyzed static, and dynamic.
  - Safety and other valves are ignored
  - Welds and bolted joints are not included.
  - The manhole is modeled as a part of the tank
  - Other fittings and accessories are not considered
  - In fatigue analysis, the effect of pressure variation caused by sloshing is ignored because it is small and occurring for a short time.
  - Leakage from joint fail is not considered.

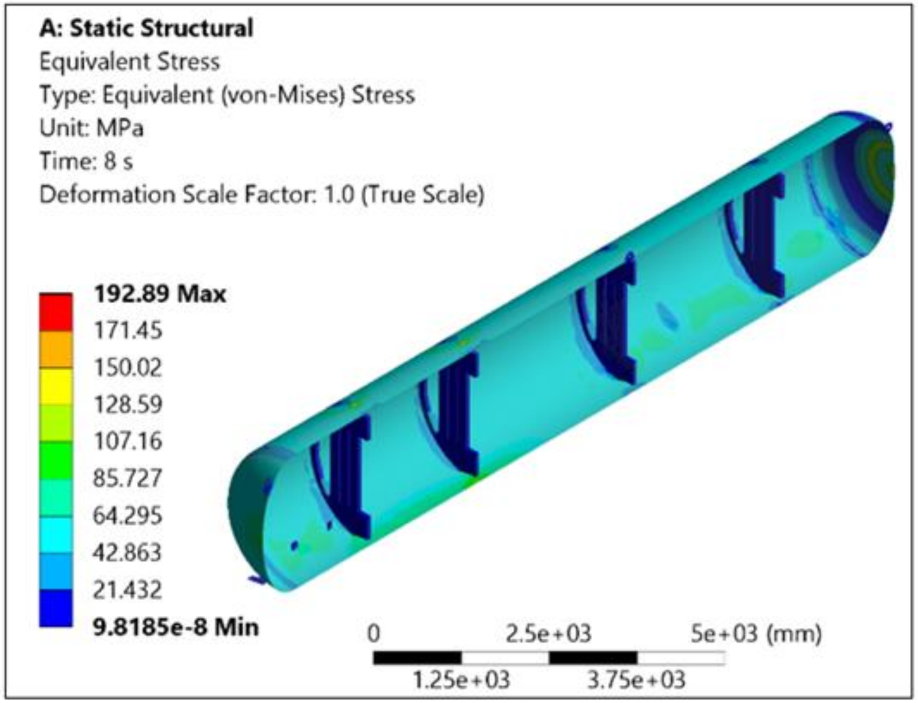
- The mesh is 179590 elements
- The element size is 10 mm at the manhole and is 80 mm for the rest of the tank
- Half the tank is modeled (planar symmetry)

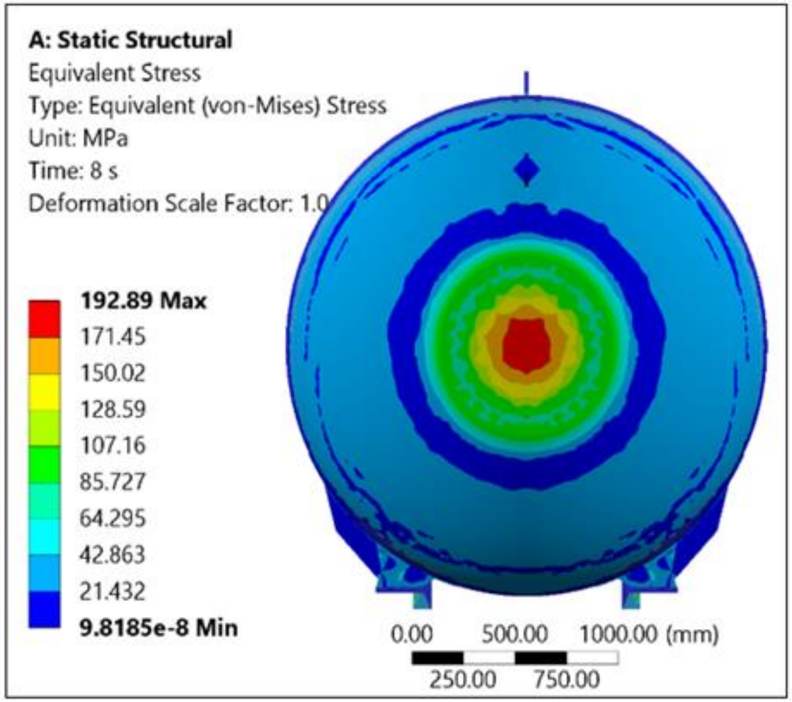


- Static stress Analysis
  - A symmetry boundary conditions are applied along the symmetric plane
  - A 90.4 psi pressure is applied inside the vessel walls
  - The pressure generate hoop and axial stresses in the tank
- Fatigue stress Analysis
  - A repeated pressure of 90.4 psi (0.623 MPa) is applied inside the tank as shown
  - The endurance limit is 116.5 MPa

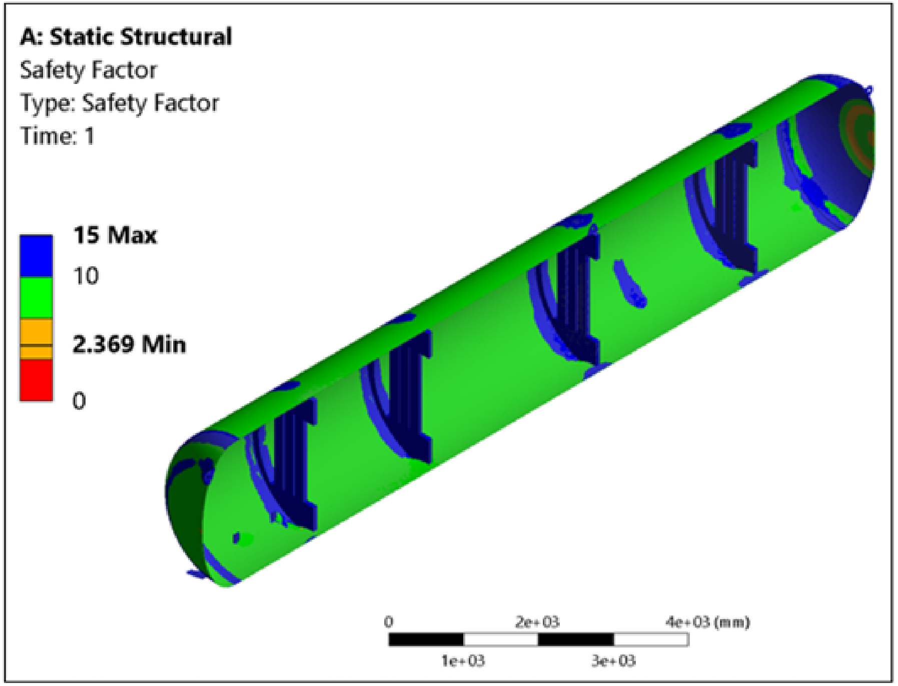


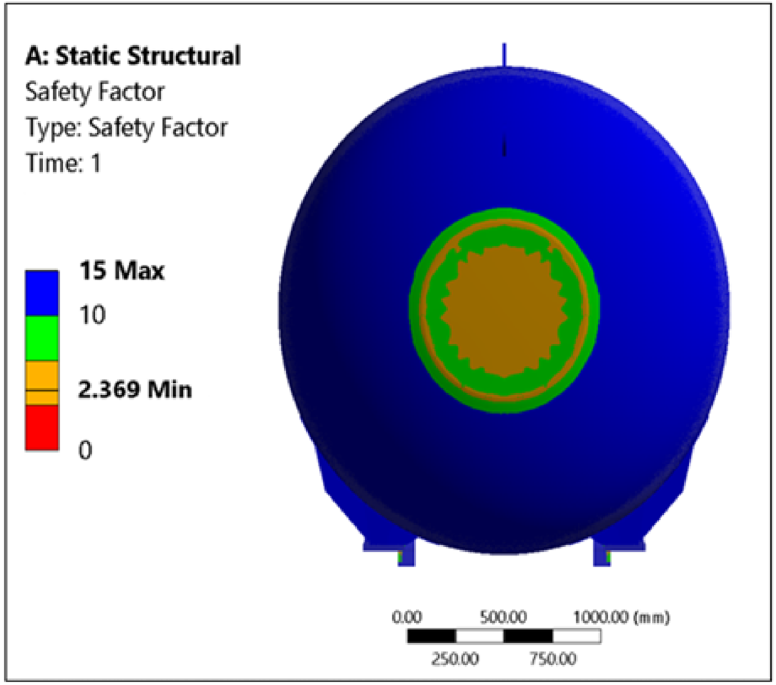
- Von-Mises stresses (static, 97% filling)



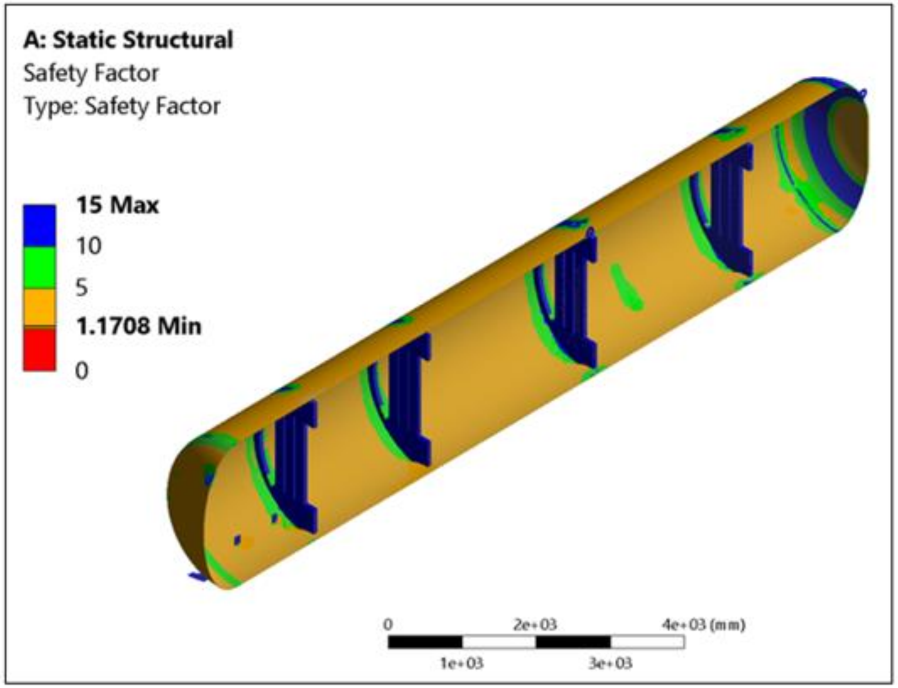


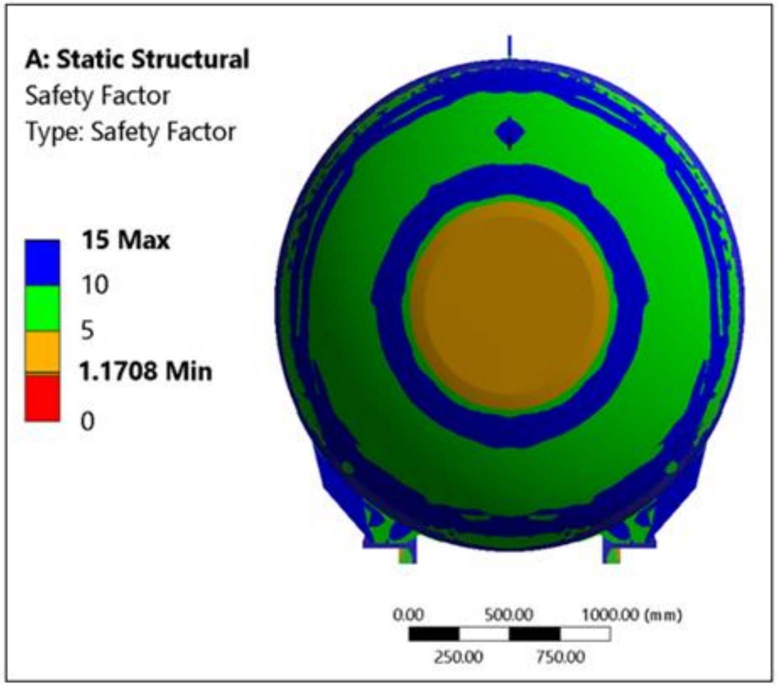
- Factor of safety (static, 97% filling)





- Factor of safety (Fatigue, 97% filling)





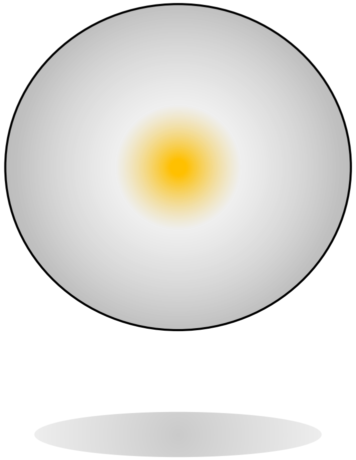
- The factor of safety for different filling of the tank are shown in Table below.

Filling	97%	90%	80%	70%	50%
Static	2.37	2.41	2.46	2.52	2.68
Fatigue	1.17	1.19	1.22	1.25	1.32

4.2.7 Conclusions from the Stress Analyses

- When the tank is filled with 97%, both static and fatigue factor of safeties are safe.
- However, all the joints and their components must be analyzed to make sure that they will remain safe with the pressure caused by large filling.
- Numerical simulations help minimizing the number of tests required that are very costly and time consuming,
- Once simulations are validated by test results, it can be used as design tool for the improvement of the system structure involved.
- LPG can be filled up to 95%; however, all axillary components must be checked for handling the increased pressure.

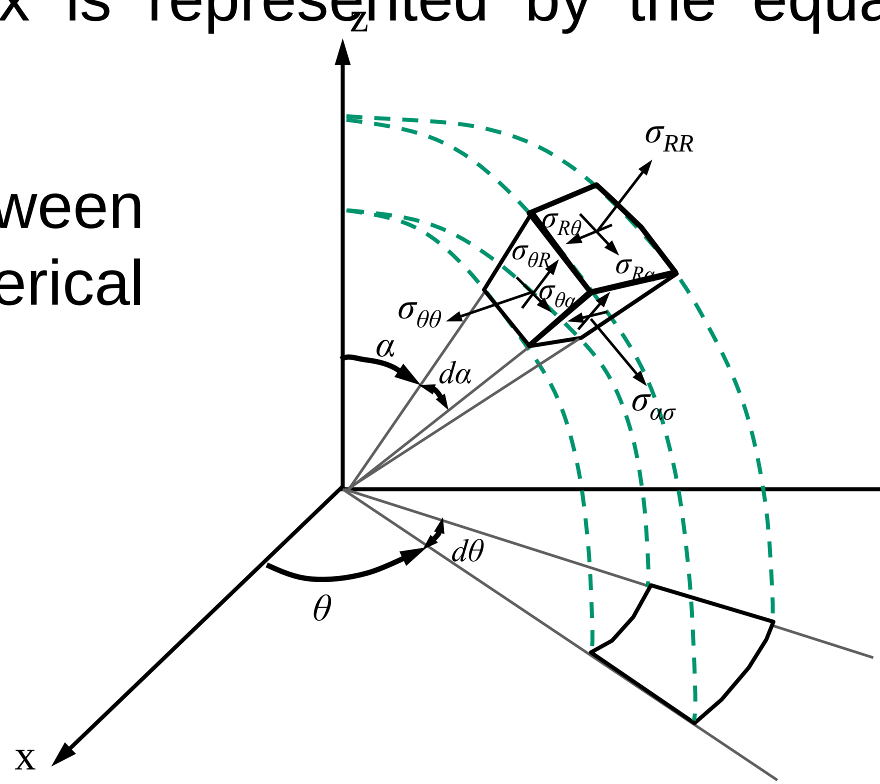
4.3 Spherical Body Subjected to Thermal Heat



4.3.1 Background

- Let us have a solid sphere that is subject to steady state heat flux. The surface of radius b is traction free (stresses at the surface are equal to zero)
- A steady state heat flux is represented by the equation $q_R = q_o(3\cos 2\alpha + 1)$
- The relation between rectangular and spherical coordinates are

$$x = R\cos\theta\sin\alpha$$
$$y = R\sin\theta\sin\alpha$$
$$Z = R\cos\alpha$$



4.3.2 Methodology

- An expression for the temperature field is determined, taking into consideration that the temperature is harmonic ($\nabla^2 T = 0$), and the relationships between the heat flux and temperature is ($q = -k\nabla T$).
- A particular thermoelastic solution is obtained using the temperature function *T*.
- Then, an appropriate spherical hormonic potentials is used in solutions above to complete the solution.
- The boundary conditions are applied to determine the multiplying constants to get the full stresses and displacements

4.3.3 Theoretical Analysis

- The relationships between heat fluxes and temperature gradients at any given point is:

$$q = -k\nabla T \tag{1}$$

or

$$q_R = -k\frac{\partial T}{\partial R},\ q_\alpha = -\frac{k}{R}\frac{\partial T}{\partial \alpha},\ q_\theta = -\frac{k}{R\sin\alpha}\frac{\partial T}{\partial \theta} \tag{2}$$

- The general energy conservation equation for the sphere with time-dependent temperature variable conductivity, and internal heat generation is:

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(kR^2\frac{\partial T}{\partial R}\right) + \frac{1}{R^2\sin\alpha}\frac{\partial}{\partial \alpha}\left(k\sin\alpha\frac{\partial T}{\partial \alpha}\right) + \frac{1}{R^2\sin^2\alpha}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C\frac{\partial T}{\partial t} \tag{3}$$

- For constant K, $q \doteq 0$, steady heat flux $\rho C \frac{\partial T}{\partial t} = 0$
- Then equation 3 becomes:

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial T}{\partial R}\right) + \frac{1}{R^2 \sin\alpha}\frac{\partial}{\partial \alpha}\left(\sin\alpha \frac{\partial T}{\partial \alpha}\right) + \frac{1}{R^2 \sin^2\alpha}\frac{\partial}{\partial \theta}\left(\frac{\partial T}{\partial \theta}\right) = 0 \quad (4)$$

- The problem is axisymmetric, hence the term

$$\frac{1}{R^2 \sin^2\alpha}\frac{\partial^2}{\partial^2 \theta} = 0 \ ,$$

- The differential equation becomes:

$$\frac{\partial}{\partial R}\left(R^2 \frac{\partial T}{\partial R}\right) + \frac{1}{\sin\alpha}\frac{\partial}{\partial \alpha}\left(\sin\alpha \frac{\partial T}{\partial \alpha}\right) = 0 \qquad (5)$$

- Boundary Conditions:

$$T(R,\alpha)|_{R=0} = finite \text{ , } T(R,\alpha)|_{R=b} = f(\alpha)$$

- Since α is the only possible orthogonal direction, with the appropriate choice of separation constant, the product solution is:

$$T(R,\alpha) = \varphi(R)\theta(\alpha)$$

- The steady state temperature solution with a selected appropriate constant for the temperature equation is:

$$T = -\frac{q_o R^2}{2kb}(3\cos 2 \propto +1)$$

- The full stresses and displacement fields are:

$$\sigma_{ZZ} = \frac{2\alpha E q_o\left(Z^2+2r^2-b^2\right)}{Kb(7+5\nu)} \, , \; \sigma_{rr} = \frac{\alpha E q_o\left(Z^2-r^2+b^2\right)}{Kb(7+5\nu)} \, , \; \sigma_{\theta\theta} = \frac{\alpha E q_o\left(Z^2-5r^2+b^2\right)}{Kb(7+5\nu)} \, ,$$

$$\sigma_{rZ} = -\frac{2\alpha E q_o r Z}{Kb(7+5\nu)} \, , \; \sigma_{r\theta} = 0 \, , \; \sigma_{\theta Z} = 0$$

$$u_r = -\frac{\alpha(1+\upsilon) q_o r\left(2r^2+b^2-13Z^2\right)}{Kb(7+5\nu)} \, , \; u_z = -\frac{\alpha(1+\upsilon) q_o Z\left(4Z^2-11r^2+2b^2\right)}{Kb(7+5\nu)} \, ,$$

$$u_\theta = 0$$

- The principle stresses are:

$$\sigma_1 = \frac{\alpha E q_o}{2Kb(7+5\nu)}[3(Z^2+r^2) - b^2 + (3b^2(3b^2-2Z^2-10r^2) + r^2(25r^2+26Z^2) + Z^4)^{0.5}]$$

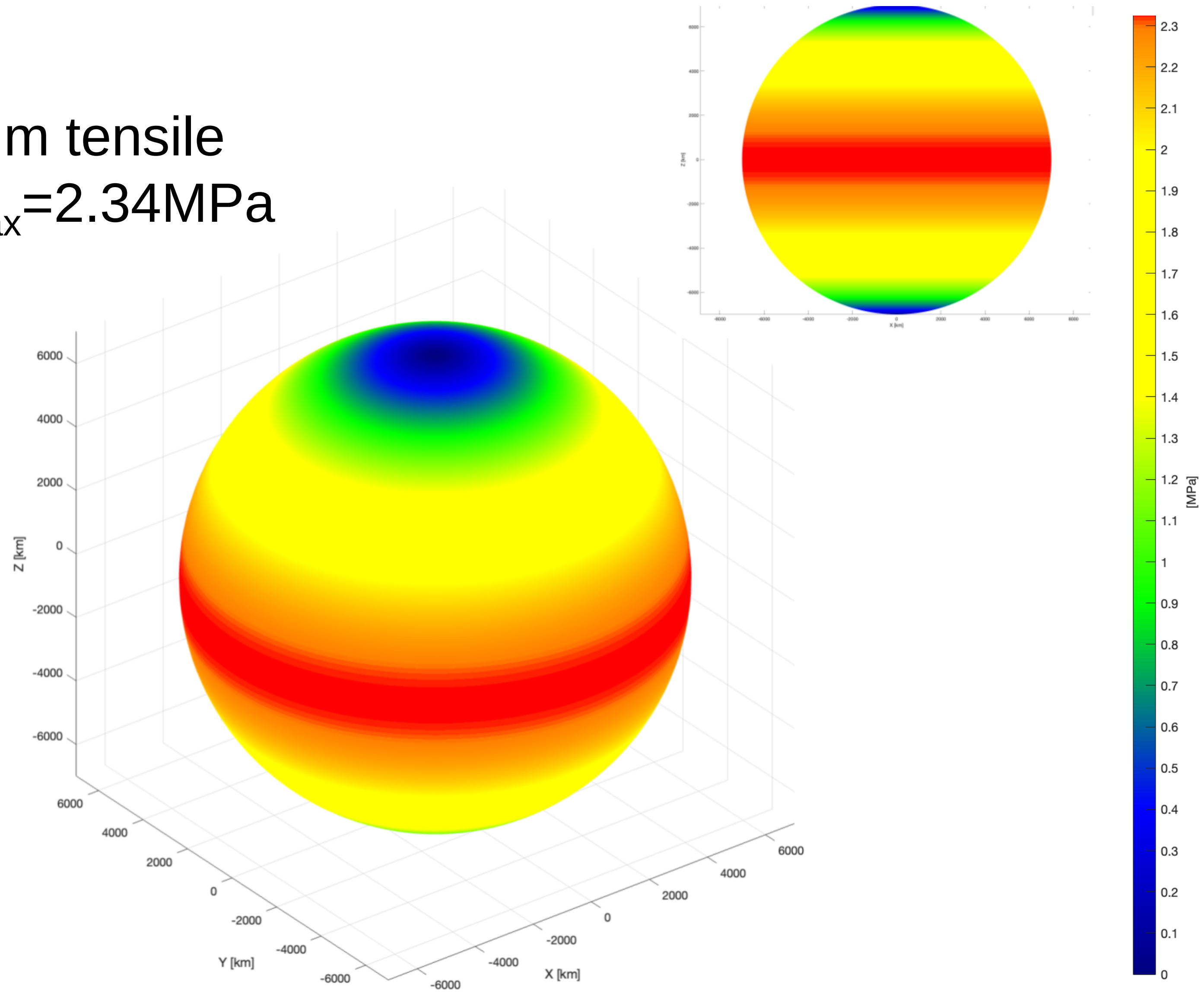
$$\sigma_2 = \frac{\alpha E q_o}{2Kb(7+5\nu)}[3(Z^2+r^2) - b^2 + (3b^2(3b^2-2Z^2-10r^2) + r^2(25r^2+26Z^2) + Z^4)^{0.5}]$$

$$\sigma_3 = \frac{\alpha E q_o}{2Kb(7+5\nu)}[Z^2 - 5r^2 + b^2]$$

4.3.4 Applied Data for the Analysis

Parameter	Data
Radius	7×10^{7} m
Yung's Modulus	49GPa
Poisson's Ratio	0.28
Coefficient of thermal expansion	6.5×10^{-6} /^{o}C
Conductivity K	2.5 W/m^{o}C
Heat Flux q_o	1.1×10^{-5} W/m^2

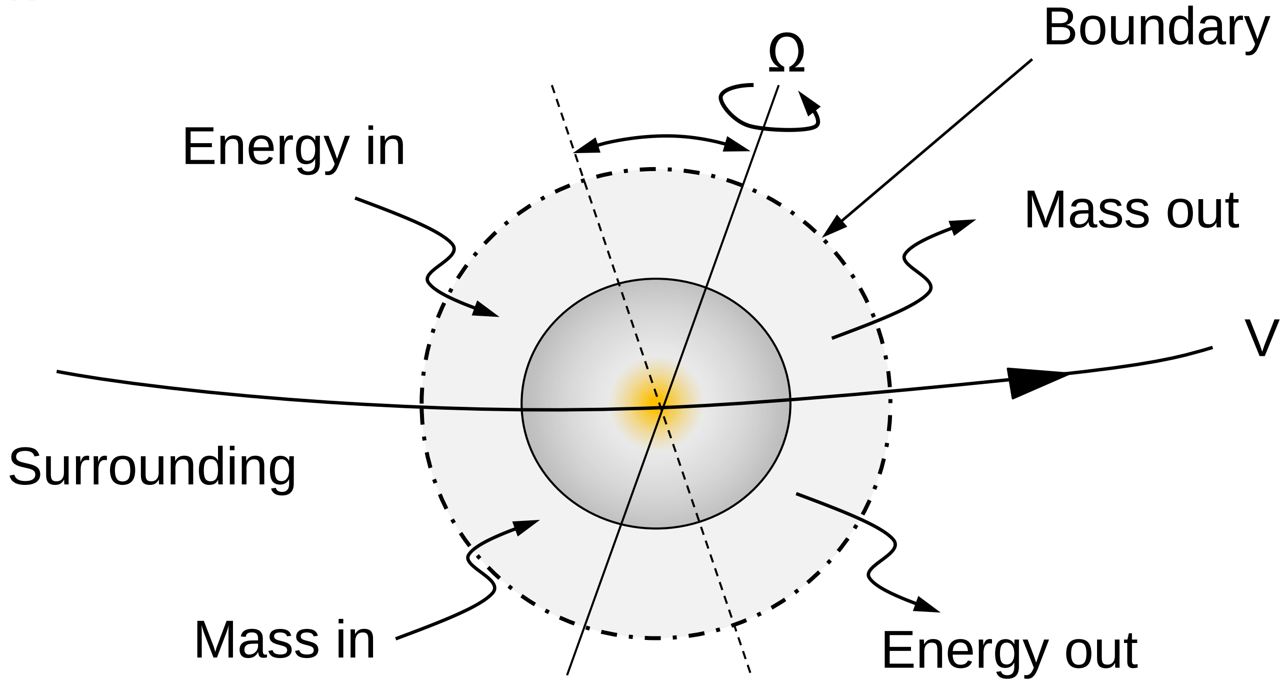
- The maximum tensile stress is σ_{max}=2.34MPa



- A theoretical method is used to find the solution to the spherical body.
- The problem is coupled between the elasticity of the body and the steady state heat flux that is subjected to.
- The surface of the sphere is traction free
- A harmonic function is created to help finding the particular thermoelastic solution T, to get the stress and displacements effected by the heat flux.

4.3.5 Effect of Adding More Variable Parameters

– The dynamic motion of the system
– The motion of the continentals
– The convection heat transfer of the air
– Other terms that may effect the solution
– Etc…



Thank You